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The calculation of ferro-concrete bridges
consisting of three spans supported on fixed piers, (1)

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Figs. 1 to 25, pp. 5 to 43.

The Bulletin of the *International Railway Congress Asssociation* of August 1927 contained an article on reinforced-concrete bridges of three spans carried on four hinged piers; the following article deals with the same type of structure modified to

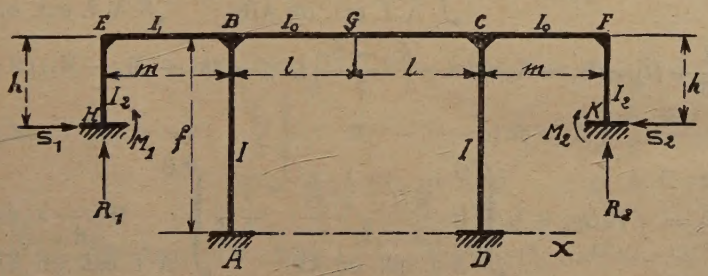

Fig. 1.

the extent that the bases of the piers are no longer hinged but are rigidly held at ADHK as shown in the diagram above (fig. 1).

Again taking $t = \frac{I_0}{I}$, $t_1 = \frac{I_0}{I_1}$, $t_2 = \frac{I_0}{I_2}$ and supposing BC to be cut at its centre G,

(1) Translated from the French.

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let us apply the two forces S, X and a moment μ at this point, as shown in figure 2, forces and moment which have to be calculated : S_1, S_2. R_1 and R_2 for the outer piers as well as the two moments M_1 and M_2 taken as acting at H and K have also to be determined.

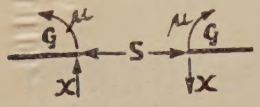

Fig. 2.

The solution of these unknowns is always obtained, as in the previous article, from the formulæ for the deflection of curved members. We shall obtain in turn, supposing as previously horizontal displacements, $\pm \Delta x_A$, $\pm e$, $\pm e'$ towards the left at A, H and K, vertical displacements $\pm \Delta y_A$, $\pm d$, $\pm d'$ downwards at A, H and K and expansion or contraction $\pm 2\lambda$ of BC, $\Delta\alpha'_0$ and $\Delta\alpha_0$ becoming nil :

$$Sf^2t + 2\mu(l+ft) - (R_1+R_2)ftm - \frac{(S_1+S_2)}{2}ft(f-2h) + (M_1+M_2)ft = Q'_1 \quad (930)$$

wherein

$$Q'_1 = I_0 \int_{A\ \text{f. ext. on ABG}}^{G} \frac{M\,ds}{I} + \frac{I_0}{I} \int_{A\ \text{f. ext. on BEH}}^{B} M\,ds$$

$$\frac{2Sf^3t}{3} + \mu f^2t - \left(\frac{R_1+R_2}{2}\right) f^2tm - \left(\frac{S_1+S_2}{6}\right) f^2t(2f-3h) + \left(\frac{M_1+M_2}{2}\right) f^2t = Q'_2 \quad (931)$$

wherein

$$Q'_2 = \mp EI_0\Delta x_A \pm 2EI_0\lambda + I_0 \int_{A\ \text{f. ext. on ABG}}^{G} \frac{M(f-y)\,ds}{I} + \frac{I_0}{I} \int_{A\ \text{f. ext. on BEH}}^{B} M(f-y)\,ds$$

$$2l^2 \times \left(\frac{l}{3} + ft\right) - (R_1-R_2)ftlm - \left(\frac{S_1-S_2}{2}\right) ftl(f-2h) + (M_1-M_2)ftl = Q'_3 \quad (932)$$

wherein

$$Q'_3 = \pm EI\Delta y_A + I_0 \int_{A\ \text{f. ext. on ABG}}^{G} \frac{M(l-x)\,ds}{I} + \frac{I_0}{I} \int_{A\ \text{f. ext. on BEH.}}^{B} M(l-x)\,ds$$

The equations for deflection following the y's and x's at H of the section ABEH and at K of the section DCFK become :

$$Sf^2t + 2\mu ft - (R_1+R_2)\,m\left(ft + \frac{mt_1}{3}\right) - \left(\frac{S_1+S_2}{2}\right)[ft(f-2h) - hmt_1] + \\ + \left(\frac{M_1+M_2}{2}\right)(2ft + mt_1) = Q'_4 \quad \ldots\ldots\ldots \quad (933)$$

wherein

$$Q'_4 = \mp \frac{EI_0}{m} \Delta y_A \pm \frac{EI_0}{m}(d + d') +$$

$$+ \frac{I_0}{I} \int_{A\ \text{f. ext. on ABG}}^{B} M dy \quad + \frac{I_0}{m} \int_{B\ \text{f. ext. on BEH}}^{H} E\ \frac{M(m-x)\,dx}{I} \quad + \frac{I_0}{I} \int_{0\ \text{f. ext. on BEH}}^{f} M ds$$

$$2ftlX - (R_1 - R_2)\frac{m}{3}(3ft + mt_1) - \left(\frac{S_1 - S_2}{2}\right)[ft(f - 2h) - hmt_1] +$$

$$+ \left(\frac{M_1 - M_2}{2}\right)(2ft + mt_1) = Q'_5 \quad . \quad . \quad . \quad . \quad . \quad . \quad (934)$$

wherein

$$Q'_5 = \mp \frac{EI_0}{m} \Delta y_A \pm \frac{EI_0}{m}(d - d') +$$

$$+ \frac{I_0}{I} \int_{A\ \text{f. ext. on ABG}}^{B} M dy \quad + \frac{I_0}{m} \int_{B\ \text{f. ext. on BEH}}^{H} E\ \frac{M(m-x)\,dx}{I} \quad + \frac{I_0}{I} \int_{A\ \text{f. ext. on BEH}}^{B} M dy$$

$$\frac{Sf^2t}{3}(2f - 3h) + \mu ft(f - 2h) - \left(\frac{R_1 + R_2}{2}\right) m(f^2t - 2fth - mht_1) -$$

$$- \frac{S_1 + S_2}{3}(h^3t_2 + f^3t + 3h^2mt_1 - 3f^2th + 3fth^2) -$$

$$- \frac{M_1 + M_2}{2}(h^2t_2 - f^2t + 2hmt_1 + 2fth) = Q'_6 \quad . \quad . \quad . \quad . \quad . \quad (935)$$

wherein

$$Q'_6 = \mp EI_0 \Delta x_A \pm EI_0(h - e') + \frac{I_0}{I} \int_{A\ \text{f. ext. on ABG}}^{B} M(f - h - y)\,dy + I_0 \int_{B\ \text{f. ext. on BEH}}^{H} E\ \frac{M(f - h - y)\,ds}{I}$$

$$Xftl(f - 2h) + \left(\frac{R_1 + R_2}{2}\right) m(hmt_1 - f^2t + 2fth) + \left(\frac{S_1 + S_2}{3}\right)(h^3t_2 + f^3t + 3h^2mt_1 - 3f^2th + 3fth^2) -$$

$$- \left(\frac{M_1 - M_2}{2}\right)(h^2t_2 - f^2t + 2hmt_1 + 2fth) = Q'_7 \quad . \quad . \quad . \quad . \quad (936)$$

wherein

$$Q'_7 = \mp EI_0 \Delta x_A \pm EI_0(l + l') + \frac{I_0}{I} \int_{A\ \text{f. ext. on ABG}}^{B} M(f - h - y)\,dy + I_0 \int_{A\ \text{f. ext. on BEH.}}^{H} E\ \frac{M(f - h - y)\,ds}{I}$$

The calculations of the angular deformation at H and K, nil deflection at these points, leads to the two equations :

$$Sf^2t + 2\mu ftl - \left(\frac{R_1 + R_2}{2}\right) m(mt_1 + 2ft) + \left(\frac{S_1 + S_2}{2}\right)(h^2t_2 - f^2t + 2hmt_1 + 2fth) + \\ + (M_1 + M_2)(ht_2 + mt_1 + ft) = Q'_8 \quad . \quad . \quad . \quad . \quad . \quad . \quad (937)$$

wherein

$$Q'_8 = \frac{I_0}{I}\int_{A\ \text{f. ext. on ABG}}^{B} M dy \qquad + I_0 \int_{A\ \text{f. ext. on BEH}}^{\substack{H\\E\\B}} \frac{M ds}{I}$$

$$- 2Xftl + \left(\frac{R_1 - R_2}{2}\right) m(mt_1 + 2ft) - \left(\frac{S_1 - S_2}{2}\right)(h^2t_2 - f^2t + 2hmt_1 + 2fth) - \\ - (M_1 - M_2)(ht_2 + mt_1 + ft) = Q'_9 \quad . \quad . \quad . \quad . \quad . \quad . \quad (938)$$

wherein

$$Q'_9 = -Q'_8 = -\frac{I_0}{I}\int_{A\ \text{f. ext. on ABG}}^{B} M dy \qquad - I_0 \int_{A\ \text{f. ext. on BEH.}}^{\substack{H\\E\\B}} \frac{M ds}{I}$$

The nine equations (930 to 938) include nine unknowns of the first degree S, μ, X, R_1, R_2, S_1, S_2, M_1, M_2, and form two systems, one including five equations 930, 931, 933, 935 and 937 with five unknowns S, μ, $R_1 + R_2$, $S_1 + S_2$, $M_1 + M_2$, and the other including four equations 932, 934, 936 and 938 with four unknowns X, $R_1 - R_2$, $S_1 - S_2$ and $M_1 - M_2$.

Solving these two systems gives, by lengthy calculations useless to reproduce here :

$$S = -\frac{A_1K'_1 + B_1K'_2}{f^2th^2t_2mt_1C_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (939)$$

$$\mu = \frac{K'_2h - 3K'_1}{C_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (940)$$

$$X = -\frac{K'_3D_1 + K_4E_1}{N_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (941)$$

$$S_1 - S_2 = \frac{+K_3D'_1 - K_4E'_1}{N_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (942)$$

$$S_1 + S_2 = \frac{2Sf^3t(3ft + mt_1) - 12\mu fl(3ft + mt_1) - 2(3fH'_1 + H'_2)}{f^3t(3ft + mt_1)} \quad . \quad . \quad . \quad . \quad . \quad (943)$$

$$M_1 + M_2 = 2Sf + \frac{4\mu[mt_1(l + ft) + 3ftl]}{ftmt_1} - (S_1 + S_2)(f + h) + \frac{2H'_1}{ftmt_1} \quad . \quad . \quad . \quad (944)$$

$$R_1 + R_2 = \frac{2Sf^2t + 4\mu(l + ft) - (S_1 + S_2)ft(f - 2h) + 2(M_1 + M_2)ft - 2Q'_1}{2ftm} \quad (945)$$

$$M_1 - M_2 = \frac{4Xl[3ftl + mt_1(3ft + l)]}{3ftmt_1} - (S_1 - S_2)(f + h) - \frac{2H'_5}{3ftmt_1} \quad . \quad . \quad . \quad . \quad (946)$$

$$R_1 - R_2 = \frac{4l(l + 3ft)X - 3(S_1 - S_2)ft(f - 2h) + 6(M_1 - M_2)ft - \frac{6}{l}Q'_3}{6mft} \quad . \quad . \quad (947)$$

in which :

$$A_1 = 3f(4ht_2 + mt_1)(4lmt_1 + ftmt_1 + 4ftl) + 6ht_2l(3hmt_1 - 2f^2t) \quad . \quad . \quad . \quad (948)$$

$$B_1 = 3fh(2ht_2 + mt_1)(4lmt_1 + ftmt_1 + 4ftl) + 6h^2t_2l(hmt_1 - f^2t) \quad . \quad . \quad . \quad . \quad . \quad (949)$$

$$C_1 = 2fh[(ht_2 + mt_1)(4lmt_1 + ftmt_1 + 4ftl) - fthl_2l] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (950)$$

$$D'_1 = l(mt_1 + 4ht_2)(lmt_1 + 3ftmt_1 + 4ftl) - 4ftl^2ht_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (951)$$

$$E'_1 = lh(mt_1 + 2ht_2)(lmt_1 + 3ftmt_1 + 4ftl) + f^2tl^2mt_1 - 2ftl^2h^2t_2 \quad . \quad . \quad . \quad . \quad (952)$$

$$D_1 = -\frac{3}{4}ftmt_1[fmt_1 + 2ht_2(2f + h)] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (953)$$

$$E_1 = -\frac{1}{4}ftmt_1[3fhmt_1 + 2h^2t_2(3f + h) - f^3l] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (954)$$

$$N_1 = \frac{ftlmt_1}{4}\{(lmt_1 + 3ftmt_1 + 4ftl)[4h^3t_2(ht_2 + mt_1) + f^3t(4ht_2 + mt_1)] + 3f^2tlmt_1(fmt_1 + 4fht_2 + 4h^2t_2) - 4ftlht_2(h^3t_2 + f^3t)\} \quad . \quad . \quad . \quad (955)$$

$$H'_1 = -(3ft + mt_1)I_0 \int_A^B \frac{M\,ds}{I} - (3ft + mt)\frac{I_0}{I}\int_0^f M\,dy +$$

(A f. ext. on ABG ; 0 f. ext on BEH)

$$+ 3ft\frac{I_0}{I}\int_A^B M\,dy + 3ft\frac{I_0}{m}\int_E^H \frac{M(m-x)}{I}\,ds + 3ft\frac{I_0}{I}\int_0^f M\,dy \mp$$

(A f. ext. on ABG ; B f. ext. on BEH ; 0 f. ext. on BEH)

$$\mp 3ft\frac{EI_0}{m}(\Delta y_A + d + d') \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (956)$$

$$H'_2 = 6(3ft + mt_1)I_0 \int_A^B \frac{M(f-y)\,ds}{I} + 6(3ft + mt_1)\frac{I_0}{I}\int_0^f M(f-y)\,dy \mp$$

(A f. ext. on ABG ; 0 f. ext. on BEH)

$$\mp 6(3ft + mt_1)EI_0(\Delta x_A - 2\lambda) - 9f^2t\frac{I_0}{I}\int_A^B M\,dy - \frac{9f^2tI_0}{m}\int_E^H \frac{M(m-x)\,ds}{I} -$$

(A f. ext. on ABG ; B f. ext. on BEH)

$$- 9f^2t\frac{I_0}{I}\int_0^f M\,dy \pm \frac{9f^2t}{m}EI_0(\Delta y_A - d - d') \quad . \quad . \quad . \quad (957)$$

(0 f. ext. on BEH)

$$H'_3 = ft\frac{I_0}{I}\int_0^f M(f - h - y)\,dy + ftI_0\int_E^H \frac{M(f-h-y)\,ds}{I} -$$

(0 f. ext. on ABG ; A f. ext. on BEH)

$$-\left(\frac{f^2t-2fth-hmt_1}{2}\right)I_0\int_{A\ \text{f. ext. on ABG}}^{G}B\ \frac{M\,ds}{I}\qquad-\left(\frac{f^2t-2fth-hmt_1}{2}\right)\frac{I_0}{I}\int_{0\ \text{f. ext. on BEH}}^{f}M\,dy\qquad\mp$$

$$\mp ft\,EI_0(\Delta x_A-e+e')\quad\ldots\ldots\ldots\ldots\quad(958)$$

$$H'_4=\left(\frac{mt_1+2ft}{2}\right)I_0\int_{A\ \text{f. ext on ABG}}^{G}B\ \frac{M\,ds}{I}\qquad+\left(\frac{mt_1+2ft}{2}\right)\frac{I_0}{I}\int_{0\ \text{f. ext. on BEH}}^{f}M\,dy\qquad+$$

$$+ft\frac{I_0}{I}\int_{0\ \text{f. ext. on ABG}}^{f}M\,dy\qquad-ftI_0\int_{A\ \text{f. ext. on BEH}}^{\substack{H\\E\\B}}\frac{M\,ds}{I}\quad\ldots\quad(959)$$

$$K'_1=f[(ht_2+mt_1)\,H'_1h+mt_1H'_3]+$$

$$+\frac{(3fH'_1+H'_2)\,mt_1}{6f(3ft+mt_1)}(6fh^2t_2+3fhmt_1+2h^3t_2-f^3t).\quad\ldots\quad(960)$$

$$K'_2=f[(mt_1+2ht_2)\,H'_1+mt_1H'_4]+\frac{mt_1(3fH'_1+H'_2)}{2f(3ft+mt_1)}(fmt_1+4fht_2+2h^2t_2)\quad(961)$$

$$H'_5=\pm\frac{3}{l}(3ft+mt_1)\,EI_0\Delta y_A\pm\frac{9ft}{m}EI_0(\Delta y_A-d+d')+$$

$$+\frac{3}{l}(3ft+mt_1)\,I_0\int_{A\ \text{f. ext. on ABG}}^{G}B\ \frac{M(l-x)\,ds}{I}\qquad+\frac{3}{l}(3ft+mt_1)\frac{I_0}{I}\int_{A\ \text{f. ext. on BEH}}^{B}M(l-0)\,ds\quad-$$

$$-9ft\frac{I_0}{I}\int_{0\ \text{f. ext. on ABG}}^{f}M\,dy\qquad-\frac{9ft\,I_0}{m}\int_{B\ \text{f. ext. on BEH}}^{\substack{H\\E}}\frac{M(m-x)\,ds}{I_1}\qquad-9ft\frac{I_0}{I}\int_{0\ \text{f. ext. on BEH}}^{f}M\,dy\qquad(962)$$

$$H'_6=ft\frac{I_0}{I}\int_{A\ \text{f. ext. on ABG}}^{B}M(f-h-y)\,dy+ftI_0\int_{A\ \text{f. ext. on BEH}}^{\substack{H\\E\\B}}\frac{M(f-h-y)\,ds}{I}-$$

$$-\left(\frac{f^2t-2fth-hmt_1}{2l}\right)I_0\int_{A\ \text{f. ext. on ABG}}^{G}B\ \frac{M(l-x)\,ds}{I}\qquad-\left(\frac{f^2t-2fth-hmt_1}{2l}\right)\frac{I^0}{I}\int_{A\ \text{f. ext. on BEH}}^{B}M(l-x)\,ds\quad-$$

$$-\left(\frac{f^2t-2fth-hmt_1}{2l}\right)EI_0\Delta y_A\mp ftEI_0(\Delta x_A-e-e')\quad\ldots\quad(963)$$

$$H'_7=\pm\left(\frac{mt_1+2ft}{2l}\right)EI_0\Delta y_A+\left(\frac{mt_1+2ft}{2l}\right)I_0\int_{A\ \text{f. ext. on ABG}}^{G}B\ \frac{M(l-x)\,ds}{I}+$$

$$+\left(\frac{mt_1+2ft}{2l}\right)\frac{I_0}{I}\int_A^B M l ds - ft\frac{I_0}{I}\int_0^f M dy - ftI_0\int_{A\,B\,E}^{H} \frac{M ds}{I} \qquad (964)$$

(A f. ext. on BEH; 0 f. ext on ABG; A f. ext. on BEH)

$$K'_3 = h(ht_2+mt_1)H'_5 - 3mt_1H'_6 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (965)$$

$$K'_4 = (2ht_2+mt_1)H'_5 - 3mt_1H'_7 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (966)$$

With these data we can start the definite calculation of all the unknowns for the various cases of loading proposed.

First case. — Local vertical load P_1 on the middle span acting at a horizontal distance a from B, a being $\leqslant l$.

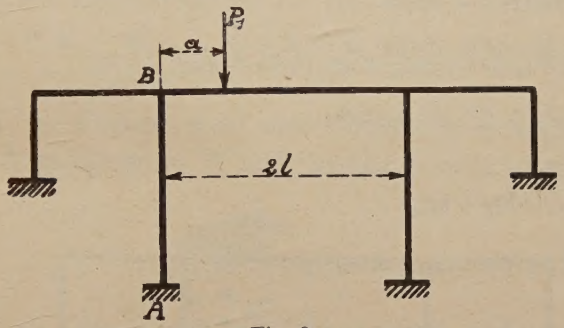

Fig. 3.

We shall have :

$$H'_1 = -\frac{P_1 a}{2}[a(3ft+mt_1)+2ftmt_1] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (967)$$

$$H'_2 = 3P_1 af^2tmt_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (968)$$

$$H'_3 = \frac{P_1 a}{4}[aft(2h-f)+hmt_1(2ft+a)] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (969)$$

$$H'_4 = \frac{P_1 a}{4}[a(2ft+mt_1)+2ftmt_1] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (970)$$

$$H'_5 = \frac{P_1 a}{2l}[a(3l-a)(3ft+mt_1)+6ftlmt_1] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (971)$$

$$H'_6 = \frac{P_1 a}{12l}[a(3l-a)(2fth+hmt_1-f^2t)+6ftlmt_1] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (972)$$

$$H'_7 = \frac{P_1 a}{12l}[a(3l-a)(2ft+mt_1)+6ftlmt_1] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (973)$$

$$K'_1 = -P_1 a(\alpha'_1+\alpha'_2 a) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (974)$$

wherein

$$\alpha'_1 = \frac{f^2thmt_1}{2}(2ht_2+mt_1) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (975)$$

$$\alpha'_2 = \frac{1}{2}[fh^2t_2(3ft + 4mt_1) + hmt_1(2f^2t + 2fmt_1 + h^2t_2)] \quad . \quad . \quad . \quad . \quad . \quad (976)$$

$$K'_2 = -P_1 a(\alpha'_3 + \alpha'_4 a) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (977)$$

wherein

$$\alpha'_3 = \frac{f^2tmt_1}{2}(4ht_2 + mt_1) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (978)$$

$$\alpha'_4 = \frac{1}{2}[2fht_2(3ft + 4mt_1) + mt_1(2f^2t + 2fmt_1 + 3h^2t_2)] \quad . \quad . \quad . \quad . \quad (979)$$

$$K'_3 = P_1 a(\alpha'_5 + 3\alpha'_6 al - \alpha'_6 a_2) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (980)$$

wherein

$$\alpha'_5 = \frac{3}{2}fthmt_1(2ht_2 + mt_1) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (981)$$

$$\alpha'_6 = \frac{1}{4l}[2h^2t_2(3ft + mt_1) + mt_1(4fth + hmt_1 + f^2t)] \quad . \quad . \quad . \quad . \quad . \quad . \quad (982)$$

$$K'_4 = P_1 a(\alpha'_7 + 3\alpha'_8 al - \alpha'_8 a^2) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (983)$$

wherein

$$\alpha'_7 = \frac{3}{2}ftmt_1(4ht_2 + mt_1) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (984)$$

$$\alpha'_8 = \frac{1}{4l}[4ht_2(3ft + mt_1) + mt_1(4ft + mt_1)] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (985)$$

$$Q'_1 = \frac{P_1 a}{2}(a + 2ft) \quad (986) \qquad Q'_3 = \frac{P_1 a}{6}[6ftl + a(3l - a)] \quad . \quad . \quad . \quad . \quad (987)$$

Introducing these quantities into formulæ (939) to (946) we obtain :

$$S = \frac{3P_1 a(2l - a)mt_1(2h^2t_2 + 2hmt_1 - f^2t)}{2C_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (988)$$

$$\mu = P_1 a(\alpha'_9 + \alpha'_{10} a) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (989)$$

wherein

$$\alpha'_9 = \frac{f^2thmt_1}{C_1}(ht_2 + mt_1)(990) \text{ et } \alpha'_{10} = \frac{fh}{2C_1}[ht_2(3ft + 4mt_1) + 4mt_1(ft + mt_1) \quad (991)$$

$$S_1 + S_2 = -\frac{3P_1 a(2l - a)f^2tmt_1}{C_1}(992) \quad M_1 + M_2 = \frac{P_1 a(2l - a)f^2thmt_1}{C_1} \quad . \quad . \quad (993)$$

$$R_1 + R_2 = -\frac{3P_1 af^2th}{mC_1}(2l - a)(ht_2 + 2mt_1) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (994)$$

$$X = P_1 a[\alpha'_{13} + \alpha'_{14} a(3l - a)] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (997)$$

wherein

$$\alpha'_{13} = +\frac{3}{8}\frac{f^2t^2m^2t_1^2l}{lN_1}[f^3t(4ht_2 + mt_1) + 4h^3t_2(ht_2 + mt_1)] \quad . \quad . \quad . \quad . \quad . \quad (998)$$

$$\alpha'_{14} = +\frac{1}{16}\frac{ftmt_1}{lN_1}\{(mt_1 + 4ft)[f^3t(4ht_2 + mt_1) + 4h^3t_2(ht_2 + mt_1)] + 3f^2tmt_1(fmt_1 + 4fht_2 + 4h^2t_2) - 4fthl_2(h^3t_2 + f^3t)\} \quad . \quad . \quad . \quad (999)$$

When $a = 0, X = 0$

$$S_1 - S_2 = -\frac{3 P_1 a}{4 N_1} f^2 l^2 m^2 t_1^2 (f m t_1 + 2 h^2 t_2 + 4 f h t_2)(l - a)(2 l - a) \quad . \quad . \quad . \quad . \quad (1000)$$

When $a = 0, S_1 - S_2 = 0$

$$M_1 - M_2 = \frac{P_1 a}{4 N_1} f^2 t^2 m^2 t_1^2 (2 h^3 t_2 + 6 f h^2 t_2 + 3 f h m t_1 - f^3 t)(l - a)(2 l - a) \quad . \quad . \quad (1001)$$

$$R_1 - R_2 = -\frac{3 P_1 a f^2 t^2 m t_1 (l - a)(2 l - a)}{4 m N_1} [f^3 t (2 h t_2 + m t_1) + h^2 t_2 (2 h^2 t_2 + 3 f m t_1 + 4 h m t_1)] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1002)$$

These formulæ are easily simplified for the case $a = l$ when it will be seen that $X = \frac{P}{2}$, $S_1 - S_2 = 0$, $M_1 - M_2 = 0$, $R_1 - R_2 = 0$, which proves the exactitude thereof.

Second case. — Vertical load distributed over a length a of the centre span, starting from the point B, and varying from p_1 to p_2, a being $\leqslant l$.

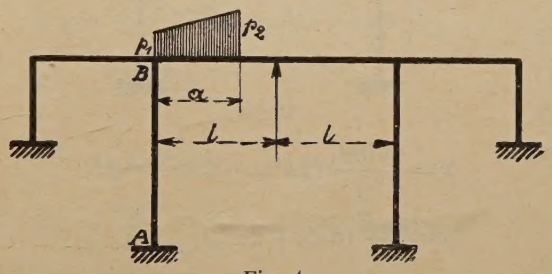

Fig. 4.

Applying the preceding case, we obtain readily :

$$S = m t_1 \frac{a^2}{8 C_1} (2 h^2 t_2 + 2 h m t_1 - f^2 t)[p_1 (4 l - a) + p_2 (8 l - 3 a)] \quad . \quad . \quad . \quad (1003)$$

$$\mu = \frac{a^2}{12} [p_1 (2 \alpha'_9 + \alpha'_{10} a) + p_2 (4 \alpha'_9 + 3 \alpha'_{10} a)] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1004)$$

$$S_1 + S_2 = -\frac{a^2}{4 C_1} f^2 t m t_1 [p_1 (4 l - a) + p_2 (8 l - 3 a)] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1005)$$

$$M_1 + M_2 = \frac{a^2 f^2 t h m t_1}{12 C_1} [p_1 (4 l - a) + p_2 (8 l - 3 a)] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1006)$$

$$R_1 + R_2 = -\frac{a^2 f^2 t h (h t_2 + 2 m t_1)}{4 m C_1} [p_1 (4 l - a) + p_2 (8 l - 3 a)] \quad . \quad . \quad . \quad . \quad . \quad . \quad (1007)$$

$$X = \frac{a^2}{60} \{p_1 [10 \alpha'_{13} + 3 \alpha'_{14} (5 l - a)] + p_2 [20 \alpha'_{13} + 3 \alpha'_{14} (15 l - 4 a)]\} \quad . \quad (1008)$$

$$S_1 - S_2 = -\frac{a^2 f^2 l^2 m^2 t_1^2}{80 N_1}(f m t_1 + 2 h^2 t_2 + 4 f h t_2)[p_1(20 l^2 - 15 a l + 3 a^2) + p_2(40 l^2 - 45 a l + 12 a^2)] \quad (1009)$$

$$M_1 - M_2 = \frac{a^2 f^2 l^2 m^2 t_1^2}{240 N_1}(2 h^3 t_2 + 6 f h^2 t_2 + 3 f h m t_1 - f^3 t)[p_1(20 l^2 - 15 a l + 3 a^2) + p_2(40 l^2 - 45 a l + 12 a^2)] \quad (1010)$$

$$R_1 - R_2 = -\frac{a^2 f^2 l^2 m t_1}{80 m N_1}[f^3 t(2 h t_2 + m t_1) + h^2 t_2(2 h^2 t_2 + 3 f m t_1 + 4 h m t_1)][p_1(20 l^2 - 15 a l + 3 a^2) + p_2(40 l^2 - 45 a l + 12 a^2)] \quad (1011)$$

Note. — If in the case we have just considered $p_1 = p_2$ and $a = l$, we obtain (see fig. 5) :

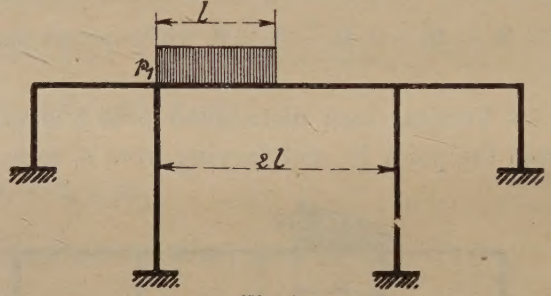

Fig. 5.

$$S = \frac{p_3 l^3 m t_1}{C_1}(2 h^2 t_2 + 2 h m t_1 - f^2 l) \quad (1012)$$

$$\mu = \frac{p_1 l^2}{6}(3 \alpha'_9 + 2 \alpha'_{10} l) \quad (1013)$$

$$S_1 + S_2 = -\frac{2 p_1 l^3}{C_1} f^2 t m t_1 \quad (1014)$$

$$M_1 + M_2 = \frac{2 f^2 t h m t_1}{3 C_1} p_1 l^3 \quad (1015)$$

$$R_1 + R_2 = -\frac{2 p_1 l^3}{m C_1} f^2 t h (h t_2 + 2 m t_1) \quad (1016)$$

$$X = \frac{p_1 l^2}{4}(2 \alpha'_{13} + 3 \alpha'_{14} l) \quad (1017)$$

$$S_1 - S_2 = -\frac{3 p_1 l^2 f^2 t^2 m^2 t_1^2}{16 N_1}(f m t_1 + 2 h^2 t_2 + 4 f h t_2) \quad (1018)$$

$$M_1 - M_2 = \frac{p_1 l^2 f^2 t^2 m^2 t_1^2}{16 N_1}(2 h^3 t_2 + 6 f h^2 t_2 + 3 f h m t_1 - f^3 t) \quad (1019)$$

$$R_1 - R_2 = -\frac{3 p_1 l^2 f^2 t^2 m t_1}{16 N_1 m}[f^3 t(2 h t_2 + m t_1) + h^2 t_2(2 h^2 t_2 + 3 f m t_1 + 4 h m t_1)] \quad (1020)$$

THIRD CASE. — Horizontal local load P_3 acting on the upright AB at a vertical height b above the point A.

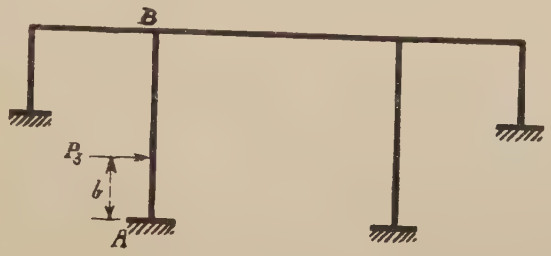

Fig. 6.

We shall have :

$$H'_1 = -\frac{P_3 b^2 t m t_1}{2} \qquad (1021)$$

$$H'_2 = \frac{P_3 b^2 t}{2}[3f(3ft + 2mt_1) - 2b(3ft + mt_1)] \qquad (1022)$$

$$H'_3 = \frac{P_3 b^2 t}{12}[ft(3f - 2b) + 3hmt_1] \qquad (1023)$$

$$H'_4 = \frac{P_3 b^2 t m t_1}{4} \qquad (1024)$$

$$H'_5 = \frac{3}{2} P_3 b^2 t m t_1 \qquad (1025)$$

$$H'_6 = \frac{P_3 b^2 t}{12}[ft(3f - 2b) + 3hmt_1] = H'^3 \qquad (1026)$$

$$H'_7 = \frac{P_3 b^2 t m t_1}{4} = H'_4 \qquad (1027)$$

$$K'_1 = P_3 b^2(\gamma'_1 + b\gamma'_2) \qquad (1028)$$

wherein

$$\gamma'_1 = \frac{htmt_1}{2}[f(2ht_2 + mt_1) + h^2 t_2] \qquad (1029)$$

$$\gamma'_2 = -\frac{htmt_1}{6f}[3f(2ht_2 + mt_1) + 2h^2 t_2] \qquad (1030)$$

$$K'_2 = P_3 b^2(\gamma'_3 + b\gamma'_4) \qquad (1031)$$

wherein

$$\gamma'_3 = \frac{tmt_1}{2}[f(4ht_2 + mt_1) + 3h^2 t_2] \qquad (1032)$$

$$\gamma'_4 = -\frac{tmt_1}{2f}[f(4ht_2 + mt_1) + 2h^2 t_2] \qquad (1033)$$

$$K'_3 = P_3 b^2 t(\gamma'_5 + b\gamma'_6) \qquad (1034)$$

wherein

$$\gamma'_5 = \frac{3mt_1}{4}\left[h(2ht_2 + mt_1) - f^2 l\right] \quad (1035)$$

$$\gamma'_6 = + \frac{f l m t_1}{2} \quad (1036)$$

$$K'_4 = P_3 b^2 t \gamma'_7 \quad (1037)$$

wherein

$$\gamma'_7 = \frac{3}{4} m t_1 (4ht_2 + mt_1) \quad (1038)$$

$$Q'_3 = \frac{P_3 b^2 t l}{2} \quad (1039)$$

$$S = \frac{P_3 b^2 (\gamma'_8 f + b\gamma'_9)}{2f^2 C_1} \quad (1040)$$

wherein

$$\gamma'_8 = 6\left[h(ht^2 + mt_1)(2lmt_1 + ftmt_1 + 4ftl) - ftl(h^2 t_2 - fmt_1)\right] \quad (1041)$$

$$\gamma'_9 = -2\left[2h(ht_2 + mt_1)(lmt_1 + ftmt_1 + 4ftl) - ftl(2h^2 t_2 - 3fmt_1)\right] \quad (1042)$$

For $b = f, \quad S = \frac{P_3}{2}$

$$\mu = -\frac{P_3 b^2 t (f-b)(ht_2 + mt_1) hmt_1}{C_1} \quad (1043)$$

$$X = \frac{3 P_3 b^2 t f t m^2 t^2_1}{8 N_1} (\gamma'_{10} + \gamma'_{11} b) \quad (1044)$$

wherein

$$\gamma'_{10} = 2h^3 t_2 (ht_2 + mt_1) - f^2 t (4fht_2 + fmt_1 + 3h^2 t_2) \quad (1045)$$

$$\gamma'_{11} = ft\left[fmt_1 + 2ht_2(2f + h)\right] \quad (1046)$$

$$S_1 - S_2 = -\frac{P_3 b^2 t f t m t_1 l}{4 N_1} (\gamma'_{12} + \gamma'_{13} b) \quad (1047)$$

wherein

$$\gamma'_{12} = 3\left[f(4ht_2 + mt_1)(2lmt_1 + 3ftmt_1 + 4ftl) + 2ht_2 l(hmt_1 - 2f^2 t)\right] \quad (1048)$$

$$\gamma'_{13} = -2\left[(mt_1 + 4ht_2)(lmt_1 + 3tfmt_1 + 4ftl) - 4ftlht_2\right] \quad (1049)$$

For $b = 0, \quad S_1 - S_2 = 0$

$$S_1 + S_2 = + \frac{6 P_3 t b^2 l m t_1}{C_1} (fb) \quad (1050)$$

For $b = 0 \quad$ et $b = f, \quad S_1 + S_2 = 0$

$$M_1 + M_2 = -\frac{2 P_3 b^2 t h l m t_1 (f-b)}{C_1} \quad (1051)$$

$$R_1 + R_2 = + \frac{6 P_3 b^2 t h l (ht_2 + 2mt_1)(f-b)}{m C_1} \quad (1052)$$

$$M_1 - M_2 = \frac{P_3 b^2 t f t l m t_1}{4 N_1} (\gamma'_{17} + b\gamma'_{18}) \quad (1053)$$

wherein

$$\gamma'_{17} = 3fh(2ht_2 + mt_1)(2lmt_1 + 3ftmt_1 + 4ftl) + 2lmt_1(f^3t + h^3t_2) - 6f^2th^2t_2l \quad (1054)$$

$$\gamma'_{18} = -2[h(2ht_2 + mt)(lmt_1 + 3ftmt_1 + 4ftl) + ftl(fmt_1 - 2h^2t_2)] \quad (1055)$$

$$R_1 - R_2 = -\frac{3P_3b^2tftlmt_1}{4N_1m}(\gamma'_{19} + \gamma'_{20}b). \quad (1056)$$

wherein

$$\gamma'_{19} = 3fh^2t_2(2lmt_1 + 3ftmt_1 + 2ftl) + 2l[h^3t_2(ht_2 + 2mt_1) - f^3t(2ht_2 + mt_1)] \quad (1057)$$

$$\gamma'_{20} = -2[h^2t_2(lmt_1 + 3ftmt_1 + 2ftl) - f^2tl(2ht_2 + mt_1)] \quad (1058)$$

For $b = 0$ and $b = f$, $M_1 + M_2$ become nil as do $R_1 + R_2$.

Fourth case. — Distributed horizontal load over a vertical length b of the upright AB starting from A and varying from p_3 to p_4 (see fig. 7).

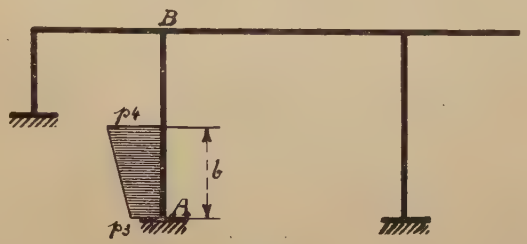

Fig. 7.

$$S = \frac{b^3}{120f^2l_1}[5f\gamma'_8(p_3 + 3p_4) + 3b\gamma'_9(p_3 + 4p_4)]. \quad (1059)$$

$$u = -\frac{b^3thmt_1(ht_2 + mt_1)}{60C_1}[5f(p_3 + 3p_4) - 3b(p_3 + 4p_4)] \quad (1060)$$

$$X = \frac{b^3tftm^2t^2_1}{160N_1}[5\gamma'_{10}(p_3 + 3p_4) + 3b\gamma'_{11}(p_3 + 4p_4)] \quad (1061)$$

$$S_1 - S_2 = -\frac{b^3tftlmt_1}{240N_1}[5\gamma'_{12}(p_3 + 3p_4) + 3b\gamma'_{13}(p_3 + 4p_4)] \quad (1062)$$

$$S_1 + S_2 = +\frac{b^3tlmt_1}{10C_1}[5f(p_5 + 3p_4) - 3b(p_3 + 4p_4)] \quad (1063)$$

$$M_1 + M_2 = -\frac{b^3thlmt_1}{30C_1}[5f(p_3 + 3p_4) - 3b(p_3 + 4p_4)] \quad (1064)$$

$$R_1 + R_2 = +\frac{b^3thl}{10mC_1}(ht_2 + 2mt_1)[5f(p_3 + 3p_4) - 3b(p_3 + 4p_4)] \quad (1065)$$

$$M_1 - M_2 = \frac{b^3tftlmt_1}{240N_1}[5\gamma'_{17}(p_3 + 3p_4) + 3b\gamma'_{18}(p_3 + 4p_4)] \quad (1066)$$

$$R_1 - R_2 = -\frac{b^3tftlmt_1}{80mN_1}[5\gamma'_{19}(p_3 + 3p_4) + 3b\gamma'_{20}(p_3 + 4p_4)] \quad (1067)$$

The values of γ'_8 to γ'_{20} are the same as these in the third case above.

We shall find quite readily the simplifications to be made in these formulæ when $b = f$.

If, in addition, from $b = f$ we take $p_3 = p_4$ we obtain (see fig. 8):

$$S = \frac{p_3 f^2}{4C_1}[2h(ht_2 + mt_1)(3lmt_1 + ftmt_1 + 4ftl) + ftl(fmt_1 - 2h^2t_2). \quad (1068)$$

$$\mu = -\frac{p_3 f^4}{12C_1} thmt_1(ht_2 + mt_1). \quad (1069)$$

$$X = \frac{p_3 f^4 l^2 m^2 t^2_1}{32N_1}(4\gamma'_{10} + 3f\gamma'_{11}) \quad (1070)$$

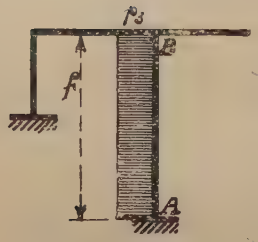

Fig. 8.

$$S_1 - S_2 = -\frac{p_3 f^4 t^2 lmt_1}{48N_1}(4\gamma'_{12} + 3f\gamma'_{13}). \quad (1071)$$

$$S_1 + S_2 = \frac{p^3 f^4 tlmt_1}{2C_1} \quad (1072)$$

$$M_1 + M_2 = -\frac{p^3 f^4 thlmt_1}{6C_1} \quad (1073)$$

$$R_1 + R_2 = +\frac{p_3 f^4 thl(ht_2 + 2mt_1)}{2mC_1} \quad (1074)$$

$$M_1 - M_2 = \frac{p_3 f^4 t^2 lmt_1}{48N_1}(4\gamma'_{17} + 3f\gamma'_{18}) \quad (1075)$$

$$R_1 - R_2 = -\frac{p_3 f^4 t^2 lmt_1}{96}(4\gamma'_{19} + 3f\gamma'_{20}) \quad (1076)$$

Fifth case. — Vertical local load P_5 acting on the extreme left hand span at a horizontal distance d from B, d being $\leqslant m$ (see fig. 9).

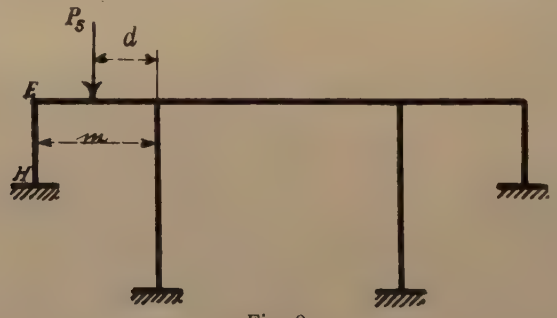

Fig. 9.

We shall have :

$$H'_1 = \frac{P_5 dftt_1}{2m}[2m^2 - d(3m - d)] \quad (1077)$$

$$H'_2 = \frac{3P_5 d f^2 t t_1}{2m}[d(3m-d) - 2m^2]. \quad (1078)$$

$$H'_3 = -\frac{P_5 d f t t_1 h}{2}(m-d). \quad (1079)$$

$$H'_4 = -\frac{P_5 d f t t_1}{2}(m-d) \quad (1080)$$

$$H'_5 = \frac{3P_5 d f t t_1}{2m}[d(3m-d) - 2m^2] \quad (1081)$$

$$H'_6 = -\frac{P_5 d f t t_1 h}{2}(m-d). \quad (1082)$$

$$H'_7 = -\frac{P_5 d}{2} f t t_1 (m-d) \quad (1083)$$

$$K'_1 = P_5 d(\delta'_1 + \delta'_2 d + \delta'_3 d^2). \quad (1084)$$

wherein

$$\delta'_1 = \frac{f^2 h t t_1}{2} m(2ht_2 + mt_1) \quad (1085)$$

$$\delta'_2 = -\frac{f^2 h t t_1}{2}(3ht_2 + 2mt_1) \quad (1086)$$

$$\delta'_3 = \frac{f^2 h t t_1}{2m}(ht_2 + mt_1). \quad (1087)$$

$$K'_2 = P_5 d(\delta'_4 + \delta'_5 d + \delta'_6 d^2) \quad (1088)$$

$$\delta'_4 = f^2 t t_1 m \frac{(4ht_2 + mt_1)}{2} \quad (1089)$$

$$\delta'_5 = -f^2 t t_1 (3ht_2 + mt_1). \quad (1090)$$

$$\delta'_6 = \frac{f^2 t t_1}{2m}(2ht_2 + mt_1). \quad (1091)$$

$$K'_3 = -P_5 d(\delta'_7 + \delta'_8 d + \delta'_9 d^2) \quad (1092)$$

$$\delta'_7 = \frac{3 f t t_1 h m}{2}(2ht_2 + mt_1). \quad (1093)$$

$$\delta'_8 = -\frac{3 f t t_1 h}{2}(3ht_2 + 2mt_1) \quad (1094)$$

$$\delta'_9 = \frac{3 f t t_1 h}{2m}(ht_2 + mt_1). \quad (1095)$$

$$K'_4 = -\frac{3P_5 d t_1 f t}{2m}[m^2(4ht_2 + mt_1) - 2mt(3ht_2 + mt_1) + d^2(2ht_2 + mt_1)] \quad (1096)$$

which gives :

$$S = -\frac{3P_5 d t_1}{2mC_1}(m^2\delta'_{10} + md\delta'_{11} + d^2\delta'_{12}) \quad (1097)$$

wherein $\delta'_{10} = 2l(2ht^2t_2 + 2hmtt_1 - f^2t)$ . . . . . (1098)

$\delta'_{11} = -f(4lmt_1 + ftmt_1 + 4ftl) - 2l(3h^2t_2 + 4lmt_1 - 2f^2l)$ . . (1099)

$\delta'_{12} = f(4lmt_1 + ftmt_1 + 4ftl) + 2l(h^2t_2 + 2hmt_1 - f^2t)$ . . . . (1100

It will be seen that for $d = 0$ and $d = m$, $S = 0$

$$u = -\frac{P_5 dt_1 f^2 th}{2mC_1}[2(mt_1 + ht_2)m^2 - md(3ht_2 + 4mt_1) + d^2(ht_2 + 2mt_1)] \quad (1101)$$

$$X = -\frac{3P_5 dt_1 f^2 t^2 mt}{8mN_1}(m^2\delta'_{15} + md\delta'_{16} + d^2\delta'_{17}) \quad (1102)$$

wherein

$\delta'_{15} = 4h^3t_2(ht_2 + mt_1) + f^3t(4ht_2 + mt_1)$ . . . . . (1103)

$\delta'_{16} = -[2h^2t_2(3ht_2 + 4mt_1) + 2f^3t(3ht_2 + mt_1) + 3fh^2t_2mt_1]$ . . . . (1104)

$\delta'_{17} = 2h^3t_2(ht_2 + 2mt_1) + f^3t(2ht_2 + mt_1) + 3fh^2t_2mt_1$ . . . . . (1105)

For $d = 0$ and $d = m$, $X = 0$

$$S_1 - S_2 = \frac{3P_5 dftt_1 lmt_1}{2mN_1}(m^2\delta'_{18} + md\delta'_{19} + d^2\delta'_{20}) \quad (1106)$$

wherein $\delta'_{18} = ftl(2h^2t_2 + 4fht_2 + fmt_1)$ . . . . . (1107)

$\delta'_{19} = [mt_1h^2t_2(l + 3ft) - 2f^2tl(3ht_2 + mt_1)]$ . . . . . (1108)

$\delta'_{20} = -[h^2t_2(lmt_1 + 3ftmt_1 + 2ftl) - f^2tl(2ht_2 + mt_1)]$ . . . . (1109)

For $d = 0$ and $d = m$, $S_1 - S_2 = 0$

$$S_1 + S_2 = +\frac{3P_5 dt_1}{mC_1}(m^2\delta'_{21} + md\delta'_{22} + d^2\delta'_{23}) \quad (1110)$$

wherein $\delta'_{21} = 2f^2tl$ . . . . . (1111)

$\delta'_{22} = f(4lmt_1 + ftmt_1 + 4ftl) - 4f^2tl$ . . . . . (1112)

$\delta'_{23} = -f(4lmt_1 + ftmt_1 + 4ftl) + 2f^2tl$ . . . . (1113)

It will be seen that for $d = 0$ and $d = m$, $S_1 + S_2 = 0$

$$M_1 + M_2 = -\frac{P_5 dt_1 fh}{mC_1}(m^2\delta'_{24} + md\delta'_{25} + d^2\delta'_{26}) \quad (1114)$$

wherein $\delta'_{24} = 2ftl$ . . . . . (1115)

$\delta'_{25} = 4lmt_1 + ftmt_1$ . . . . . (1116)

$\delta'_{26} = -(4lmt_1 + ftmt_1 + 2ftl)$ . . . . . (1117)

It will be seen that for $d = 0$ and $d = m$, $M_1 + M_2 = 0$

$$R_1 + R_2 = \frac{P_5 dt_1 fh}{m^3 t_1 C_1}(m^2\delta'_{27} + md\delta'_{28} + d^2\delta'_{29}) \quad (1118)$$

wherein $\delta'_{27} = 6ftl(ht_2 + 2mt_1)$ . . . . . (1119)

$\delta'_{28} = 3(ht_2 + 2mt_1)(4lmt_1 + ftmt_1)$ . . . . . (1120)

$\delta'_{29} = -(ht_2 + 4mt_1)(4lmt_1 + ftmt_1) - 4ftlmt_1$ . . (1121)

For $d = 0$, $R_1 + R_2 = 0$ and for $d = m$, $R_1 + R_2 = P_1$

$$M_1 - M_2 = \frac{P_5 dt_1 ftlmt_1}{4mN_1}(m^2\delta'_{30} + md\delta'_{31} + d^2\delta'_{32}) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1122)$$

wherein

$$\delta'_{30} = -2(3fhmt_1 + 6fh^2t_2 + 2h^3t_2 - f^3t)\,ftl \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1123)$$

$$\delta'_{31} = [mt_1(l + 3ft)(f^3t - 2h^3t_2) + 3f^2tl(fmt_1 + 4hmt_1 + 6h^2t_2)] \quad . \quad . \quad . \quad . \quad (1124)$$

$$\delta'_{32} = -[(f^3t - 2h^3t_2)(lmt_1 + 3ftmt_1 + 2ftl) + 3f^3tl(fmt_1 + 2hmt_1 + 2h^2t_2)] \quad . \quad (1125)$$

For $d = 0$ and for $d = m$, $M_1 - M_2 = 0$

$$R_1 - R_2 = \frac{P_5 dt_1 ftl}{4m^2N_1}(m^2\delta'_{33} + md\delta'_{34} + d^2\delta'_{35}) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1126)$$

wherein

$$\delta'_{33} = +6ftl[f^3t(2ht_2 + mt_1) + 2h^3t_2(ht_2 + 2mt_1) + 3fh^2t_2mt_1)] \quad . \quad . \quad . \quad . \quad (1127)$$

$$\delta'_{34} = +6mt_1\{(l + 3ft)[f^3t(2ht_2 + mt_1) + 2h^3t_2(ht_2 + mt_1)] + 3f^2tl(fmt_1 + 2fht_2 - 2h^2t_2)\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1128)$$

$$\delta'_{35} = -2mt_1\{(l + 3ft)[f^3t(ht_2 + mt_1) + h^3t_2(ht_2 + 4mt_1)] + ftl(f^3t + 3f^2mt_1 + 3f^2ht_2 - 6fh^2t_2)\} \quad . \quad . \quad (1129)$$

For $d = 0$, $R_1 - R_2 = 0$. For $d = m$, $R_1 - R_2 = P_5$.

Sixth case. — Vertical load distributed over a horizontal length d calculated starting from the point B towards E and varying from p_5 to p_6 (see fig. 10).

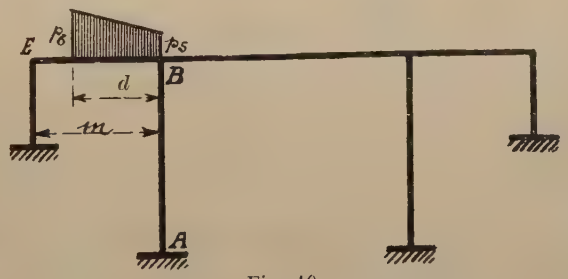

Fig. 10.

By integrating the formulæ of the preceding case, we get :

$$S = -\frac{d^2t_1}{40mC_1}[5m^2\delta'_{10}(2p_5 + 4p_6) + 5md\delta'_{11}(p_5 + 3p_6) + 3d^2\delta'_{12}(p_5 + 4p_6)] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1130)$$

$$\mu = \frac{d^2t_1f^3th}{120mC_1}[20m^2(ht_2 + mt_1)(p_5 + 2p_6) - 5md(3ht_2 + 4mt_1)(p_5 + 3p_6) + 3d^2(ht_2 + 2mt_1)(p_5 + 4p_6) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1131)$$

$$X = -\frac{d^2t_1f'^2l^2mt_1}{160mN_1}[10m^2\delta'_{15}(p_5+2p_6)+5md\delta'_{16}(p_5+3p_6)+ \\ +3d^2\delta'_{17}(p_5+4p6)] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1132)$$

$$S_1 - S_2 = \frac{d^2t_1f'tlmt_1}{40mN_1}[10m^2\delta'_{18}(p_5+2p_6)+5md\delta'_{19}(p_5+3p_6)+3d^2\delta'_{20}(p_5+4p_6)] \quad (1133)$$

$$S_1 + S_2 = \frac{d^2t_1}{20mC_1}[10m^2\delta'_{21}(p_5+2p_6)+5md\delta'_{22}(p_5+3p_6)+3d^2\delta'_{23}(p_5+4p_6)] \quad (1134)$$

$$M_1 + M_2 = -\frac{d^2t_1f'h}{60mC_1}[10m^2\delta'_{24}(p_5+2p_6)+5md\delta'_{25}(p_5+3p_6)+3d^2\delta'_{26}(p_5+4p_6)] \quad (1135)$$

$$R_1 + R_2 = \frac{d^2t_1f'h}{60m^3C_1}[10m^2\delta'_{27}(p_5+2p_6)+5md\delta'_{28}(p_5+3p_6)+3d^2\delta'_{29}(p_5+4p_6)] \quad (1136)$$

$$M_1 - M_2 = \frac{d^2t_1f'tlmt_1}{240mN_1}[10m^2\delta'_{30}(p_5+2p_6)+5md\delta'_{31}(p_5+3p_6)+3d^2\delta'_{32}(p_5+4p_6)] \quad (1137)$$

$$R_1 - R_2 = \frac{d^2t_1f'tl}{240m^2N_1}[10m^2\delta'_{33}(p_5+2p_6)+5md\delta'_{34}(p_5+3p_6)+3d^2\delta'_{35}(p_5+4p_6)] \quad (1138)$$

The δ' have the same values as in the fifth case above.

There will be no difficulty in finding the reductions to be made in the case in which $d = m$.

If furthermore from $d = m$ we have $p_5 = p_6$ (see case of fig. 11), we get :

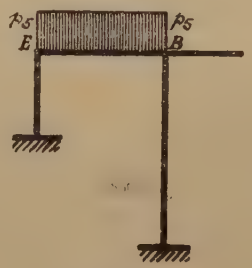

Fig. 11.

$$S = \frac{p_5m^3t_1}{3C_1}[f'(4lmt_1+ftmt_1+4ftl)-2l(3h^2t_2+2hmt_1-f^2t)] \quad . \quad . \quad (1139)$$

$$\mu = -\frac{p_5m^3t_1f'^2th}{24C_1}(3ht_2+2mt_1) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1140)$$

$$S_1 + S_2 = \frac{p_5m^3t_1}{4C_1}[f'(4lmt_1+ftmt_1+4ftl)+2f^2tl] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1141)$$

$$M_1 + M_2 = -\frac{p_5m^3t_1f'h}{12C_1}(4lmt_1+ftmt_1+6ftl) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1142)$$

$$R_1 + R_2 = \frac{p_5mf'h}{4C_1}[3ht_2+4mt_1)(4lmt_1+ftmt_1+4ftl)+4ftlmt_1] \quad . \quad . \quad . \quad . \quad . \quad (1143)$$

$$X = -\frac{p_5 m^4 t^2_1 f^2 t^2}{32 N_1}(6\delta'_{15} + 4\delta'_{16} + 3\delta'_{17}) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1144)$$

$$S_1 - S_2 = \frac{p_5 m^4 t^2_1 f t l}{8 N_1}(6\delta'_{18} + 4\delta'_{19} + 3\delta'_{20}) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1145)$$

$$M_1 - M_2 = \frac{p_5 m^4 t^2_1 f t l}{48 N_1}(6\delta'_{30} + 4\delta'_{31} + 3\delta'_{32}) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1146)$$

$$R_1 - R_2 = \frac{p_5 m^2 t_1 f t l}{48 N_1}(6\delta'_{33} + 4\delta'_{34} + 3\delta'_{35}) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1147)$$

Seventh case. — Fixed horizontal load P_6 applied at a vertical distance g from the point E of the upright EH (see fig. 12).

Fig. 12.

We shall have :

$$H'_1 = -\frac{P_6}{2} f t t_1 m (g + f) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1148)$$

$$H'_2 = P_6 \frac{f^2 t}{2}[3f^2 t + m t_1 (4f + 3g)] \quad . \quad . \quad . \quad . \quad . \quad . \quad (1149)$$

$$H'_3 = \frac{P_6 f t}{12}[f(f^2 t + 3hmt_1) + 6hmt_1 g + 2g^2 t_2 (3h - g)] \quad . \quad . \quad . \quad . \quad (1150)$$

$$H'_4 = \frac{P_6 f t}{4}(f m t_1 + 2mt_1 g + 2g^2 t_2) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1151)$$

$$H'_5 = \frac{3}{2} P_6 f t m t_1 (f + g) = -3H'_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1152)$$

$$H'_6 = \frac{P_6 f t}{12}[f(f^2 t + 3hmt_1) + 6hmt_1 g + 2g^2 t_2 (3h - g)] = H'_3 \quad . \quad (1153)$$

$$H'_7 = \frac{P_6 f t}{4}(f m t_1 + 2mt_1 g + 2g^2 t_2) = H'_4 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1154)$$

$$K'_1 = \frac{P_6}{6} f^2 t m t_1 t_2 (h^3 - 3h^2 g + 3hg^2 - g^3) = \frac{P_6}{6} f^2 t m t_1 t^2 (h - g)^3 \quad . \quad (1155)$$

$$K'_2 = \frac{P_6}{2} f^2 t m t_1 t_2 (g^2 - 2gh + h^2) = \frac{P_6}{2} f^2 t m t_1 t_2 (h - g)^2 \quad . \quad . \quad . \quad (1156)$$

$$K'_3 = \frac{P_6}{4} ftmt_1 [f(6h^2t_2 + 3hmt_1 - f^2tl) + 6h^2t_2g - 6ht_2g^2 + 2g^3t_2] \quad (1157)$$

$$K'_4 = \frac{3P_6 ftmt_1}{4} [f(4ht_2 + mt_1) + 4ht_2g - 2g^2t_2] \quad (1158)$$

$$S = -\frac{P_6}{2h^2C_1} (h^3\delta'_1 + h^2g\delta'_2 + hg^2\delta'_3 + g^3\delta'_4) \quad (1159)$$

wherein

$$\delta'_1 = 2f [(ht_2 + mt_1)(4lmt_1 + ftmt_1 + 4ftl) - fthlt_2l] \quad (1160)$$

$$\delta'_2 = -3 [fmt_1 (4lmt_1 + ftmt_1 + 4ftl) - 2h^2t_2lmt_1] \quad (1161)$$

$$\delta'_3 = -6 [fht_2 (4lmt_1 + ftmt_1 + 4ftl) + ht_2 (2hmt_1 - f^2tl)] \quad (1162)$$

$$\delta'_4 = f(4ht_2 + mt_1)(4lmt_1 + ftmt_1 + 4ftl) + 2ht_2l(3hmt_1 - 2f^2tl) \quad (1163)$$

For $g = 0$, $S = \frac{P_6}{2}$. For $g = h$, $S = 0$.

$$\mu = \frac{P_6gt_2f^2tmt_1 (h-g)^2}{2C_1} \quad (1164)$$

For $g = 0$ and for $g = h$, $\mu = 0$.

$$S_1 + S_2 = -\frac{P_6g}{h^2C_1} (h^2\delta'_5 + gh\delta'_6 + g^2\delta'_7) \quad (1165)$$

wherein

$$\delta'_5 = 3fmt_1 (4lmt_1 + ftmt_1 + 4ftl) \quad (1166)$$

$$\delta'_6 = 6 [fht_2 (4lmt_1 + ftmt_1 + 4ftl) - ht_2lf^2tl)] \quad (1167)$$

$$\delta'_7 = -[(4fht_2 + fmt_1)(4lmt_1 + ftmt_1 + 4ftl) - 4f^2thlt_2l] \quad (1168)$$

For $g = 0$, $S_1 + S_2 = 0$ and for $g = h$, $S_1 + S_2 = -P_6$.

$$M_1 + M_2 = \frac{P_6g}{h^2C_1} (h^2\delta'_8 + hg\delta'_9 + g^2\delta'_{10}) \quad (1169)$$

wherein

$$\delta'_8 = fhmt_1 (4lmt_1 + ftmt_1 + 4ftl) \quad (1170)$$

$$\delta'_9 = 2fh^2t_2 (4lmt_1 + ftmt_1 + 4ftl) - 2f^2t_2h^2t_2l \quad (1171)$$

$$\delta'_{10} = -fh (mt_1 + 2ht_2)(4lmt_1 + ftmt_1 + 4ftl) + 2fth^2t_2l \quad (1172)$$

For $g = 0$ and $g = h$, $M_1 + M_2 = 0$.

$$R_1 + R_2 = \frac{3P_6gt_2f}{mC_1} (4lmt_1 + ftmt_1 + 2ftl)(h-g)^2 \quad (1173)$$

For $g = 0$ and $g = h$, $R_1 + R_2 = 0$.

$$S_1 - S_2 = -\frac{P_6ftmt_1l}{4N_1} (f^2t\delta'_{11} + 6ht_2g\delta'_{12} + 6t_2g^2\delta'_{13} - 2g^3t_2\delta'_{14}) \quad (1174)$$

wherein

$$\delta'_{11} = f(4ht_2 + mt_1)(lmt_1 + 3ftmt_1 + 4ftl + 2lht_2(3mt_1h - 2f^2t) + 3flmt_1 (4ht_2 + mt_1) \quad (1175)$$

$$\delta'_{12} = mt_1 [h(lmt_1 + 3ftmt_1 + 4ftl) + 2f^2tl] \quad (1176)$$

$$\delta'_{13} = 2h^2t_2 (lmt_1 + 3ftmt_1 + 4ftl) - ftl(fmt_1 + 2h^2t_2) \quad (1177)$$

$$\delta'_{14} = (mt_1 + 4ht_2)(lmt_1 + 3ftmt_1 + 4ftl) - 4ftlht_2 \quad (1178)$$

For $g = h, \qquad S_1 - S_2 = -P_6.$

$$X = \frac{3P_6 f^2 l^2 m^2 l^2_1}{8N_1}\left[h^2 t_2 \delta'_{15} + hgt_2\delta'_{16} + g^2 t_2 \delta'_{17} + g^3 t_2 \delta'_{18}\right] \quad . \quad . \quad (1179)$$

wherein

$$\delta'_{15} = 2fh\,(ht_2 + mt_1) - f^3 t \quad . \quad . \quad (1180)$$

$$\delta'_{16} = 2\,(h^3 t_2 + f^3 t) - 3fhmt_1 \quad . \quad . \quad (1181)$$

$$\delta'_{17} = -\left[4h^3 t_2 + f^3 t + 6fh^2 t_2\right] \quad . \quad . \quad (1182)$$

$$\delta'_{18} = f\,(4ht_2 + mt_1) + 2h^2 t_2 \quad . \quad . \quad (1183)$$

For $g = h, \qquad X = 0.$

$$M_1 - M_2 = \frac{P_6 ftlmt_1}{4N_1}\left(f^2 th\delta'_{19} + mt_1 g\delta'_{20} + g^2 t_2 \delta'_{21} + g^3 t_2 \delta'_{22}\right) \quad . \quad . \quad (1184)$$

wherein

$$\delta'_{19} = f\,(2ht_2 + mt_1)\,(4lmt_1 + 3ftmt_1 + 4ftl) + 2lht_2\,(hmt_1 - f^2 t) \quad . \quad . \quad (1185)$$

$$\delta'_{20} = (2h^3 t_2 - f^3 t)\,(lmt_1 + 3ftmt_1 + 4ftl) + 3f^2 tl\,(2h^2 t_2 - fmt_1) \quad . \quad . \quad (1186)$$

$$\delta'_{21} = 2\left[(2h^3 t_2 - f^3 t)\,(lmt_1 + 3ftmt_1 + 3ftl) - 3f^2 tlmt_1\,(f+h)\right. \quad . \quad . \quad (1187)$$

$$\delta'_{22} = -2h\,(2ht_2 + mt_1)\,(lmt_1 + 3ftmt_1 + 4ftl) + 2ftl\,(2h^2 t_2 + fmt_1) \quad . \quad . \quad (1188)$$

For $g = h, \qquad M_1 - M_2 = 0.$

$$R_1 - R_2 = \frac{3}{4}\frac{P_6 ftlmt_1}{mN_1}\left(\delta'_{23} + g\delta'_{24} + g^2 t_2 \delta'_{25} + g^3 t_2 \delta'_{26}\right) \quad . \quad . \quad (1189)$$

wherein

$$\delta'_{23} = -f^3 th^2 t_2\,(lmt_1 + 3ftmt_1 + 2ftl) - f^2 th^2 t_2 l\,(2h^2 t_2 + 3fmt_1 + 4hmt_1) \quad (1190)$$

$$\delta'_{24} = 2ht_2\,(h^3 t_2 + f^3 t)\,(lmt_1 + 3ftmt_1 + 2ftl) + 6f^2 tht_2 lmt_1\,(f+h) \quad . \quad . \quad (1191)$$

$$\delta'_{25} = -\,(lmt_1 + 3ftmt_1 + 2ftl)\,(f^3 t + 4h^3 t_2) + 3lf^2 t\,(2h^2 t_2 - 3fmt_1) \quad . \quad . \quad (1192)$$

$$\delta'_{26} = 2h^2 t_2\,(lmt_1 + 3ftmt_1 + 2ftl) - 2f^2 tl\,(2ht_2 + mt_1) \quad . \quad . \quad (1193)$$

For $g = h, \qquad R_1 - R_2 = 0.$

Eighth case. — Horizontal load distributed over a length g of the upright EH starting from the point E and varying from p_7 to p_8 (see fig. 13).

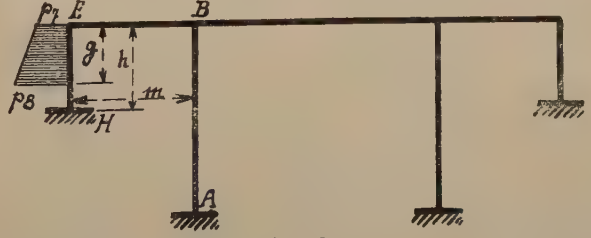

Fig. 13.

We shall readily have by the preceding case :

$$S = \frac{g}{120h^2 C_1}\left[30\delta'_1 h^3\,(p_7 + p_8) + 10h^2 g\delta'_2\,(p_7 + 2p_8) + 5hg^2\delta'_3\,(p_7 + 3p_8) + \right.$$
$$\left. + 3g^3\delta'_4\,(p_7 + 4p_8)\right] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1194)$$

$$\mu = \frac{g^2 t_2 f^2 l m t_1}{120 C_1} [10h^2 (p_7 + 2p_8) - 10hg (p_7 + 3p_8) + 3g^2 (p_7 + 4p_8)] \quad . \quad (1195)$$

$$S_1 + S_2 = -\frac{g^2}{60h^2 C_1} [10h^2 \delta'_5 (p_7 + 2p_8) + 5hg\delta'_6 (p_7 + 3p_8) + 3g^2 \delta'_7 (p_7 + 4p_8)] \quad (1196)$$

$$M_1 + M_2 = \frac{g^2}{60h^2 C_1} [10h^2 \delta'_8 (p_7 + 2p_8) + 5hg\delta'_9 (p_7 + 3p_8) + 3g^2 \delta'_{10} (p_7 + 4p_8)]. \quad (1197)$$

$$R_1 + R_2 = \frac{g^2 t_2 f}{20 m C_1} (4lmt_1 + ftmt_1 + 2ftl) [10h^2 (p_7 + 2p_8) - 10hg (p_7 + 3p_8) + 3g^2 (p_7 + 4p_8)] \quad . \quad . \quad . \quad . \quad . \quad . \quad (1198)$$

$$X = \frac{g t_2 f^2 t^2 m^2 t^2_1}{160 N_1} [30\delta'_{15} h^2 (p_7 + p_8) + 10\delta'_{16} hg (p_7 + 2p_8) + 5g^2 \delta'_{17} (p_7 + 3p_8) + 3g^3 \delta'_{18} (p_7 + 4p_8)] \quad . \quad . \quad . \quad . \quad (1199)$$

$$S_1 - S_2 = -\frac{gftlmt_1}{240 N_1} [30f^2 t\delta'_{11} (p_7 + p_8) + 60ht_2 \delta'_{12} g (p_7 + 2p_8) + 30g^2 t_2 \delta'_{13} (p_7 + p_8) - 6g^3 t_2 \delta'_{14} (p_7 + 4p_8)]. \quad . \quad . \quad . \quad (1200)$$

$$M_1 - M_2 = \frac{gftlmt_1}{240 N_1} [30f^2 th\delta'_{19} (p_7 + p_8) + 10gmt_1 \delta'_{20} (p_7 + 2p_8) + 5g^2 t_2 \delta'_{21} (p_7 + 3p_8) + 3g^3 t_2 \delta'_{22} (p_7 + 4p_8)]. \quad . \quad . \quad . \quad . \quad (1201)$$

$$R_1 - R_2 = \frac{gftlmt_1}{80 m N_1} [30\delta'_{23} (p_7 + p_8) + 10g\delta'_{24} (p_7 + 2p_8) + 5g^2 t_2 \delta'_{25} (p_7 + 3p_8) + 3g^3 t_2 \delta'_{26} (p_7 + 4p_8)] \quad . \quad . \quad . \quad . \quad . \quad (1202)$$

The values of the δ' are the same as in the seventh case.

There will be no difficulty in finding the reductions to be made in these formulæ when $g = h$.

If in addition to $g = h$ we make $p_7 = p_8$, that is to say if the arrangement is that of figure 14, we get :

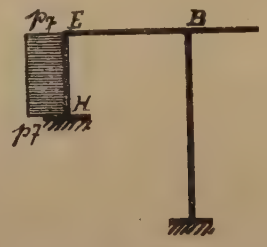

Fig. 14.

$$S = \frac{p_7 h^2}{8 C_1} [f (4lmt_1 + ftmt_1 + 4ftl) (4ht_2 + 3mt_1) + 2ht_2 l (hmt_1 - 2f^2 t)] . \quad (1203)$$

$$\mu = \frac{p_7 h^4 t_2 f^2 t m t_1}{24 C_1} \quad . \quad . \quad . \quad . \quad . \quad .

$$R_1 + R_2 = \frac{h^4 t_2 f p_7}{4mC_1}(4lmt_1 + ftmt_1 + 2ftl) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1207)$$

$$X = p_7 \frac{h^3 t_2 f^2 t^2}{32N_1} m^2 t^2{}_1 [2(h^3 t_2 - 2f^3 t) + 3fh(3mt_1 + 4ht_2)] \quad . \quad . \quad . \quad . \quad . \quad (1208)$$

$$S_1 - S_2 = -\frac{p_7 fthlmt_1}{8N_1}\{[2f^3 t(4ht_1 + mt_1) + h^3 t_2(4ht_2 + 5mt_1)](lmt_1 + 3ftmt_1 + 4ftl) + 6f^3 tlmt_1(4ht_2 + mt_1) - 4fth t_2 l(2f^3 t + h^3 t_2) + 20f^2 th^2 t_2 lmt_1\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1209)$$

$$M_1 - M_2 = \frac{p_7 fthlmt_1}{24N_1}\{(lmt_1 + 3ftmt_1 + 4ftl)[2h^2 t_2(4f^3 t + h^3 t_2) + 3hmt_1(f^3 t + h^3 t_2)] + 3f^2 thlmt_1(8fht_2 + 3fmt_1 + 5h^2 t_2) - 2fth^2 t_2 l(4f^3 t + h^3 t_2)\} \quad . \quad (1210)$$

$$R_1 - R_2 = -\frac{p_7 fthlmt_1}{8mN_1}\{2h^2 t_2(f^3 t + h^3 t_2)(lmt_1 + 3ftmt_1 + 2ftl) + 3f^2 th^2 t_2 l(2h^2 t_2 + 3hmt_1 + 6fmt_1)\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1211)$$

NINTH CASE. — Horizontal displacements $\pm \Delta x_A$, $\pm e$ and $\pm e'$ (positives towards the left) respectively at A, H and K (see fig. 15).

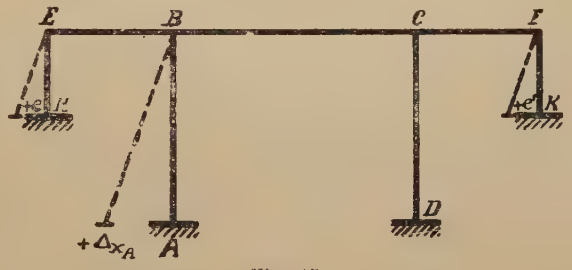

Fig. 15.

In this case :

$$H'_1 = 0 \quad . \quad . \quad . \quad (1212) \qquad H'_2 = \mp 6(3ft + mt_1)\,EI_0\,\Delta x_A \quad . \quad . \quad . \quad . \quad (1213)$$

$$H'_3 = \mp ftEI_0(\Delta x_A - e + e') \quad . \quad . \quad . \quad . \quad (1214) \qquad H'_4 = 0 \quad . \quad . \quad . \quad (1215)$$

$$K'_1 = \pm EI_0\left[-f^2 tmt_1(\Delta x_A - e + e') - 6\frac{(3ft + mt_1)\Delta x_A mt_1(6fh^2 t_2 + 3fhmt_1 + 2h^3 t_2 - f^3 t)}{6f(3ft + mt_1)}\right] =$$

$$= \mp \frac{EI_0 mt_1}{f}[\Delta x_A(6fh^2 t_2 + 3fhmt_1 + 2h^3 t^2) - f^3 t(e - e')] \quad . \quad . \quad (1216)$$

$$H'_5 = 0 \quad . \quad (1217) \qquad H'_6 = \mp ftEI_0(\Delta x_A - e - e') \quad . \quad (1218) \qquad H'_7 = 0 \quad . \quad (1219)$$

$$K'_3 = \pm 3ftmt_1\,EI_0(\Delta x_A - e - e') \quad . \quad (1220) \qquad K'_4 = 0 \quad . \quad . \quad . \quad . \quad . \quad (1221)$$

$$S = \pm \frac{3EI_0}{f^2 th^2 t_2 C_1}[\varepsilon_1 \Delta x_A - \varepsilon_2(e - e')] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1222)$$

wherein

$$\varepsilon_1 = 2h^2 t_2[lft(2h^2 t_2 - 3fmt_1) - 2h(ht_2 + mt_1)(lmt_1 + ftmt_1 + 4ftl)] \quad . \quad . \quad . \quad (1223)$$

$$\varepsilon_2 = f^2t\,[f(4ht_2 + mt_1)(4lmt_1 + ftmt_1 + 4ftl) + 2ht_2l(3hmt_1 + 2f^2t)] \quad . \quad . \quad . \quad (1224)$$

$$\mu = \pm\frac{3EI_0mt_1}{C_1}\,[2h\Delta x_A\,(ht_2 + mt_1) - f^2t\,(e - e')] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1225)$$

$$S_1 - S_2 = \pm\frac{3EI_0ftlmt_1(\Delta x_A - e - e')\left[(mt_1 + 4ht_2)(lmt_1 + 3ftmt_1 + 4ftl) - 4ftlht_2\right]}{N_1} \quad (1226)$$

$$X = \pm\frac{9f^2t^2m^2t^2_1}{4N_1}EI_0\,(\Delta x_A - e - e')\,[fmt_1 + 2ht_2\,(2f + h)] \quad . \quad . \quad . \quad . \quad . \quad (1227)$$

wherein

$$S_1 + S_2 = \pm\frac{6EI_0}{h^2t^2C_1}\{-6h^2t_2lmt_1\Delta x_A\,[f(4ht_2 + mt_1)(4lmt_1 + ftmt_1 + 4ftl) - 4f^2thtt_2l](e - e')\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1228)$$

$$M_1 + M_2 = \pm\frac{6EI_0}{ht_2C_1}\{2h^2t_2lmt_1\Delta x_A + (e - e')\,[f(2ht_2 + mt_1)(4lmt_1 + ftmt_1 + 4ftl) - 2f^2thtt_2l]\} \quad . \quad . \quad (1229)$$

$$R_1 + R_2 = \mp\frac{18EI_0}{mC_1}\,[2\Delta x_A hlmt_1 + f(e - e')(4lmt_1 + ftmt_1 + 2ftl)] \quad . \quad . \quad . \quad . \quad (1230)$$

$$M_1 - M_2 = +\frac{3EI_0ftlmt}{N_1}(\Delta x_A - e - e')\,[h\,(lmt_1 + 3ftmt_1 + 4ftl)(2ht_2 + mt_1) - ftl\,(2h^2t_2 - fmt_1)] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1231)$$

$$R_1 - R_2 = \pm\frac{9EI_0ftlmt_1}{mN_1}(\Delta x_A - e - e')\,[h^2t_2\,(lmt_1 + 3ftmt_1 + 4ftl) - ftl(fmt_1 + 2fht_2 + 2h^2t_2)] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1232)$$

Note. — The calculations of the ninth case also apply to the expansion and to the contraction of the sections EB and CF; if $+e$ and $+e'$ represent the expansions we introduce into the formulæ 1223 to 1232 $-e$ and $-e'$, and $+e'$ and $+e'$ for the contractions with $\Delta x_A = 0$.

Tenth case. — Vertical displacements $\pm\Delta y_A$, $\pm d$ and $\pm d'$ (positives towards the base) respectively at A, H and K (see fig. 16).

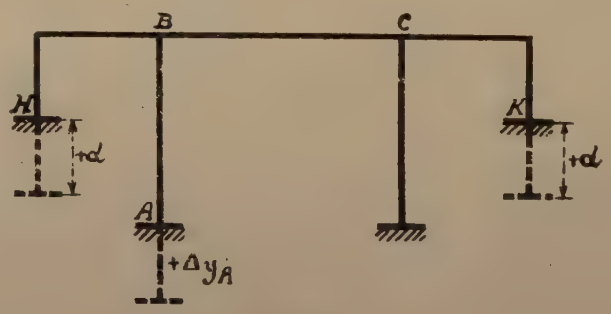

Fig. 16.

In this case :

$$H'_1 = \mp 3ft\frac{EI_0}{m}(\Delta y_A - d - d') \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1233)$$

$$H'_2 = \pm 9\frac{f^2t}{m} EI_0 (\Delta y_A - d - d') \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1234)$$

$$H'_3 = 0 \quad . \quad . \quad . \quad (1235) \qquad H'_4 = 0 \quad . \quad . \quad . \quad (1236)$$

$$K'_1 = \mp \frac{3f^2th}{m} (ht_2 + mt_1) EI_0 (\Delta y_A - d - d') \quad . \quad . \quad . \quad . \quad . \quad . \quad (1237)$$

$$K'_2 = \mp \frac{3f^2t}{m} (2ht_2 + mt_1) EI_0 (\Delta y_A - d - d') \quad . \quad . \quad . \quad . \quad . \quad . \quad (1238)$$

$$H'_5 = \pm \frac{9EI_0ft}{m} (\Delta y_A - d + d') + \frac{3}{2} (3ft + mt_1) EI_0 \Delta y_A \quad . \quad . \quad . \quad (1239)$$

$$H'_6 = \mp \frac{(f^2t - 2fth - hmt_1)}{2l} EI_0 \Delta y_A \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1240)$$

$$H'_7 = \pm \frac{(mt_1 + 2ft)}{2l} EI_0 \Delta y_A \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1241)$$

$$K'_3 = \pm EI_0 \left\{ \frac{9ft}{m} (h^2t_2 + hmt_1) (\Delta y_A - d + d') + \frac{3\Delta y_A}{2l} [2fth (3ht_2 + 2mt_1) + \right.$$
$$\left. + hmt_1 (2ht_2 + mt_1) + f^2lmt_1] \right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1242)$$

$$K'_4 = \pm EI_0 \left\{ \frac{9ft}{m} (2ht_2 + mt_1) (\Delta y_A - d + d') + \frac{3\Delta y_A}{2l} [4ft (3ht_2 + mt_1) + \right.$$
$$\left. + mt_1 (4ht_2 + mt_1)] \right. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1243)$$

$$S = \pm \frac{9EI_0\varphi'_1}{C_1} (\Delta y_A - d - d') \text{ wherein } \varphi'_1 = f(4lmt_1 + ftmt_1 + 2ftl) + 2hl(ht_2 + 2mt_1) \quad (1244)$$

$$\mu = \pm \frac{3EI_0 (\Delta y_A - d - d')}{C_1 m} (ht_2 + 2mt_1) f^2t \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1245)$$

$$S_1 - S_2 = \pm \frac{3EI_0}{N_1} [(\Delta y_A - d + d')\varphi'_2 + \Delta y_A \varphi'_3] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1246)$$

wherein

$$\varphi'_2 = \frac{3ftlmt_1}{m} [(lmt_1 + 3ftmt_1 + 4ftl) h^2t_2 - ftl^2 (2h^2t_2 + 2fht_2 + fmt_1)]. \quad (1247)$$

$$\varphi'_3 = \frac{3}{2} f^2tm^2t^2_1 (fmt_1 + 4fht_2 + 2h^2t_2) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1248)$$

$$X = \pm \frac{3EI_0ftmt_1}{4N_1} [(\Delta y_A - d + d') \varphi'_4 + \Delta y_A \varphi'_5] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1249)$$

wherein

$$\varphi'_4 = \frac{3ft}{m} [f^3t (2ht_2 + mt_1) + 2h^3t_2 (ht_2 + 2mt_1) + 3fh^2t_2mt_1] \quad . \quad . \quad . \quad (1250)$$

$$\varphi'_5 = \frac{4}{2l} [f^3tmt_1 (4ht_2 + mt_1) + h^3t_2mt_1 (ht_2 + mt_1) + f^4t^2 (3ht_2 + mt_1) +$$
$$+ fth^3t_2 (3ht_2 + 4mt_1) + 3f^2th^2t_2mt_1] \quad . \quad . \quad . \quad . \quad . \quad . \quad (1251)$$

$$S_1 + S_2 = \pm \frac{18EI_0f}{mC_1} (\Delta y_A - d - d') (4lmt_1 + ftmt_1 + 2ftl) \quad . \quad . \quad . \quad . \quad . \quad (1252)$$

$$M_1 + M_2 = \mp \frac{6EI_0 fh}{mC_1}(\Delta y_A - d - d')(4lmt_1 + ftmt_1 + 2ftl) \quad (1253)$$

$$R_1 + R_2 = \pm \frac{6EI_0 fh}{m^3 t_1 C}(\Delta y_A - d - d')[(ht_2 + 4mt_1)(4lmt_1 + ftmt_1 + 2ftl) - 2fl(ht_2 + 2mt_1)] \quad (1254)$$

$$M_1 - M_2 = \pm \frac{EI_0 l}{2N_1}[(\Delta y_A - d + d')\varphi'_6 + \Delta y_A \varphi'_7] \quad (1255)$$

wherein

$$\varphi'_6 = \frac{3ftmt_1}{m}[(lmt_1 + 3ftmt_1 + 2ftl)(f^3 t - 2h^3 t_2) + 3f^2 tl(fmt_1 + 2hmt_1 + 2h^2 t_2)] \quad (1256)$$

$$\varphi'_7 = -\frac{3}{l}[f^2 t^2 m^2 t^2_1 (6fh^2 t_2 + 3fhmt_1 + 2h^3 t_2) - f^3 t] \quad (1257)$$

$$R_1 - R_2 = \pm \frac{EI_0}{2mN_1}[(\Delta y_A + d - d')\varphi'_8 + \Delta y_A \varphi'_9)] \quad (1258)$$

wherein

$$\varphi'_8 = 2(f^3 t(ht_2 + mt_1)(4lmt_1 + 3ftmt_1) + h^3 t_2(ht_2 + 4mt_1)(lmt_1 - 3ftmt_1) - ftlmt_1(4h^3 t_2 + f^3 t)] \quad (1259)$$

$$\varphi'_9 = 9f^2 t^2 mt_1[f^3 t(2ht_2 + mt_1) + 2h^3 t_2(ht_2 + 2mt_1) + 3fh^2 t_2 mt_1] \quad (1260)$$

Eleventh case. — Expansion or contraction $\pm 2\lambda$ of the section BC.

In this case :

$$H'_1 = 0 \quad (1261) \qquad H'_2 = \pm 12EI_0\lambda(3fl + mt_1 \quad (1262)$$

$$H'_3 = 0 \quad (1263) \qquad H'_4 = 0. \quad (1265)$$

$$K'_1 = \pm \frac{2mt_1 EI_0\lambda}{f}(6fh^2 t_2 + 3fhmt_1 + 2h^3 t_2 - f^3 t) \quad (1266)$$

$$K'_2 = \pm \frac{6mt_1 EI_0\lambda}{f}(4fht_2 + fmt_1 + 2h^2 t_2) \quad (1267)$$

$$H'_5 = 0 \quad (1268) \qquad H'_6 = 0 \quad (1269) \qquad H'_7 = 0 \quad (1270)$$

$$K'_3 = 0 \quad (1271) \qquad K'_4 = 0 \quad (1272) \qquad S = \pm \frac{6EI_0\lambda\omega_1}{f^2 th^2 t_2 C_1}. \quad (1273)$$

wherein

$$\omega_1 = (4lmt_1 + ftmt_1 + 4ftl)[f^3 t(4ht_2 + mt_1) + 4h^3 t_2(ht_2 + mt_1)] - 4lht_2[ft(h^3 t_2 + f^3 t) + 3h^2 mt_1(ht_2 + mt_1) - 3f^2 thmt_1] \quad (1274)$$

$$\mu = \mp \frac{6mt_1 EI_0\lambda}{C_1}[2h(ht_2 + mt_1) - f^2 t] \quad (1275)$$

$$X = 0 \quad (1276)$$

$$S_1 - S_2 = 0 \quad (1277)$$

$$S_1 + S_2 = \pm \frac{12EI_0\lambda}{h^2 t_2 C_1}\{f(4ht_2 + mt_1)(4lmt_1 + ftmt_1 + 4ftl) - 2lht_2(2f^2 t - 3hmt_1)\} \quad (1278)$$

$$M_1 + M_2 = \pm \frac{12EI_0\lambda h}{h^2 t_2 C_1}\{f(2ht_2 + mt_1)(4lmt_1 + ftmt_1 + 4ftl - 2ht_2 l) + f^2 t - hmt_1)\} \quad (1279)$$

$$R_1 + R_2 = \pm \frac{36EI_0\lambda}{mC_1}[f(4lmt_1 + ftmt_1 + 2ftl) + 2l(h^2 t_2 + 2hmt_1)] \quad . \quad . \quad . \quad . \quad (1280)$$

$$M_1 - M_2 = 0 \quad . \quad (1281)$$

$$R_1 - R_2 = 0 \quad . \quad (1282)$$

General remarks. — I. The general remarks contained in the article on structures of the form considered above but with jointed piers. are equally applicable to the present study provided account be taken of the bases of the piers being held

II. The formulæ found in the above study are applicable to the case of the arrangement shown in sketch A (fig. 17) below, except in making in it $h = -h$, $b = -b$ and $t_2 = -t_2$.

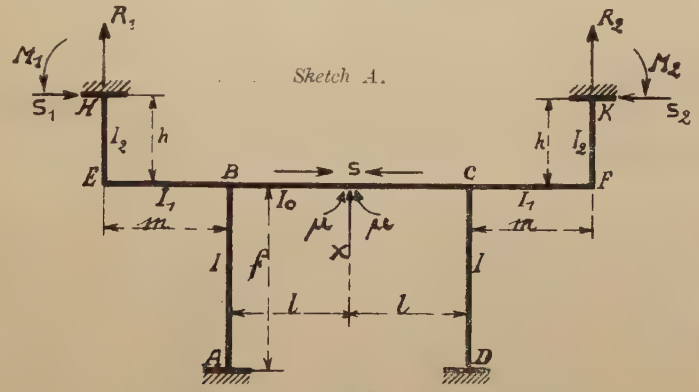

Fig. 17.

It will also be easy to calculate the form of the sketch B (fig. 18) which is only that of the sketch A turned round ; it will be sufficient to take the formulæ already found for

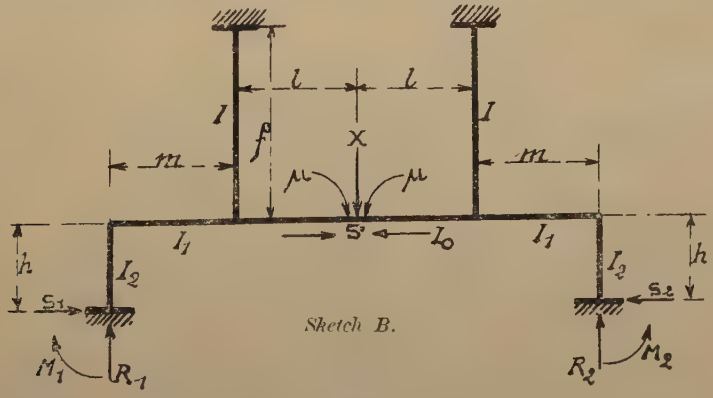

Fig. 18.

the sketch A (fig. 17) except that the positive or negative directions of the various cases of loading considered are to be taken into account.

In the same way, we can suppose in the original form of the girder studied at the beginning of the present note, that the forces act in the opposite direction to that considered; it will be sufficient in this case to reverse the signs of P_1, p_1, p_2, P_3, p_3, p_4, P_5, p_5, p_6, P_6, p_7, p_8 in the formulæ.

Furthermore, the remarks just made can be applied to the case of the structure of three spans on four piers, jointed at the base, the case studied in the note published in the August 1927 *Bulletin of the International Railway Congress Association*.

APPLICATION.

Over bridge of the same arrangement as that calculated in the preceding article for the Bruxelles-Midi to Denderleeuw line, but supposing the bases to be rigidly held instead of being jointed (see fig. 19).

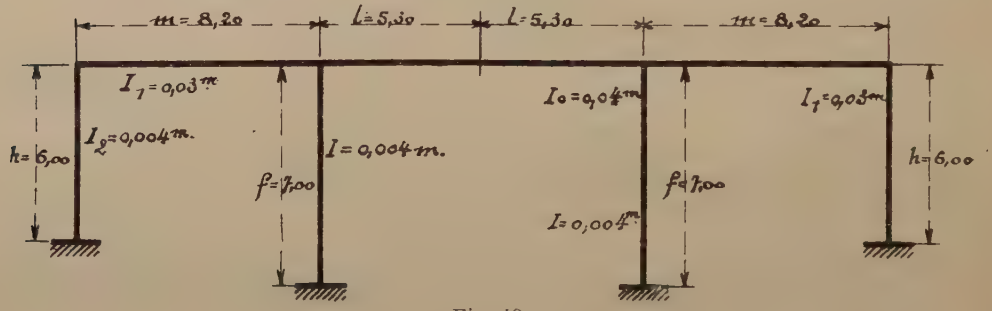

Fig. 19.

$$I_0 = 0.04m^4 \qquad I = 0.004m^4 \qquad I_1 = 0.03m^4 \qquad I_2 = 0.004m^4$$

$$t = \frac{I_0}{I} = \frac{0.04}{0.004} = 10 \qquad t_1 = \frac{I_0}{I_1} = \frac{0.04}{0.03} = \frac{4}{3} \qquad t_2 = \frac{I_0}{I_2} = \frac{0.04}{0.004} = 10$$

By formulæ 950 and 955 we get :

$$C_1 = 2 \times 7.5 \times 6\left[\left(6 \times 10 + 8\,2 \times \frac{4}{3}\right)\left(4 \times 5.3 \times \frac{4}{3} \times \right.\right.$$

$$\left.\left. \times 8.20 + 7.5 \times 10 \times \frac{4}{3} \times 8.20 + 4 \times 7.5 \times 10 \times 5.3\right) - \right.$$

$$\left. - 7.5 \times 10 \times 6 \times 10 \times 5.3\right] = 90\,(71 \times 2\,642 - 23\,850) = 14\,724\,000.$$

$$N_1 = \frac{7.5 \times 10 \times 5.3 \times 8.20 \times \frac{4}{3}}{4}\left\{\left(5.3 \times 8.2 \times \frac{4}{3} + 3 \times 7.5 \times 10 \times 8.20 \times \frac{4}{3} + \right.\right.$$

$$+ 4 \times 7.5 \times 10 \times 5.3\Big)\Big[4 \times \overline{6}^{3} \times 10\left(6 \times 10 + 8.2 \times \frac{4}{3}\right) + \overline{7.5}^{3} \times 10\Big(4 \times 6 \times 10 +$$

$$\left. + 8.2 \times \frac{4}{3}\right)\Big] + 3 \times 7.5 \times 10 \times 5.3 \times 8.2 \times \frac{4}{3}\left(7.5 \times 8.2 \times \frac{4}{3} + 4 \times 7.5 \times 6 \times \right.$$

$$\left.\left. \times 10 + 4 \times 6^2 \times 10\right) - 4 \times 7.5 \times 10 \times 5.3 \times 6.2 \times 10\left(\overline{6}^{3} \times 10 + \overline{7.5}^{3} \times 10\right)\right\} =$$

$$= 1\,084\,[4\,107\,(612\,000 + 1\,058\,900) + 97\,580 \times 3\,322 - 6\,379 \times 95\,400] =$$

$$= 1\,084 \times 6\,578\,401\,000 = 7\,130\,986\,800\,000.$$

1. *Load of 1 000 kgr. per lineal metre over the whole span BC.*

The formulæ to be used are those 1012 to 1020, except for doubling the values of S, μ, $S_1 + S_2$, $M_1 + M_2$ and $R_1 + R_2$ and cancelling X, $S_1 - S_2$, $M_1 - M_2$ and $R_1 - R_2$.

$$S = \frac{2 \times 1\,000 \times \overline{5.3}^{3} \times \frac{4}{3} \times 8.2}{14\,724\,000}\left(2 \times \overline{6}^{2} \times 10 + 2 \times 6 \times 8.2 \times \frac{4}{3} - \overline{7.5}^{2} \times 10\right) =$$

$$= \frac{56\,100 \times 58 \times 289}{14\,724\,000} = 65 \text{ kgr.}$$

$$\alpha'_9 = \frac{\overline{7.5}^{2} \times 10 \times 6 \times 8.2 \times \frac{4}{3}}{14\,724\,000}\left(60 + 8.2 \times \frac{4}{3}\right) = \frac{2\,619\,048}{14\,724\,000} = 0.1778$$

$$\alpha'_{10} = \frac{7\,5 \times 6}{2 \times 14\,724\,000}\left[60\,(3 \times 75 + 4 \times 10.93) + 4 \times 8.2 \times \frac{4}{3}\left(7.5 \times 10 + 10.93\right)\right] =$$

$$= \frac{894\,510}{2 \times 14\,724\,000} = 0.03037$$

$$\mu = \frac{2 \times 1\,000 \times \overline{5.3}^{2}}{6}(3 \times 0.1778 + 2 \times 0.03037 \times 5.3) = \frac{56180}{6} \times 0.8553 = 8\,008 \text{ Kg.-M.},$$

a figure lower than that found in the case of jointed piers, which is logical.

$$S_1 = S_2 = \frac{S_1 + S_2}{2} = -\frac{2 \times 2 \times 1\,000 \times \overline{5.3}^{3} \times \overline{7.5}^{2} \times 10 \times 8.2 \times \frac{4}{3}}{2 \times 14\,724\,000} = -$$

$$= -\frac{5.3 \times 345\,400\,258}{14\,724\,000} = -125 \text{ kgr.}$$

$$M_1 = M_2 = \frac{M_1 + M_2}{2} = \frac{2 \times 2 \times 1\,000 \times \overline{5.3}^{3} \times \overline{7.5}^{2} \times 10 \times 6 \times 8.2 \times \frac{4}{3}}{2 \times 3 \times 14\,724\,000} = 2 \times 125 = 250 \text{ Kg.-M.}$$

$$R_1 = R_2 = \frac{R_1 + R_2}{2} = -\frac{125 \times 6\,(60 + 10.93 \times 2)}{8.2 \times 10.93} = -682 \text{ kgr.}$$

The results just found enable us to prepare the diagram below (fig. 20) of the moments in every part of the structure.

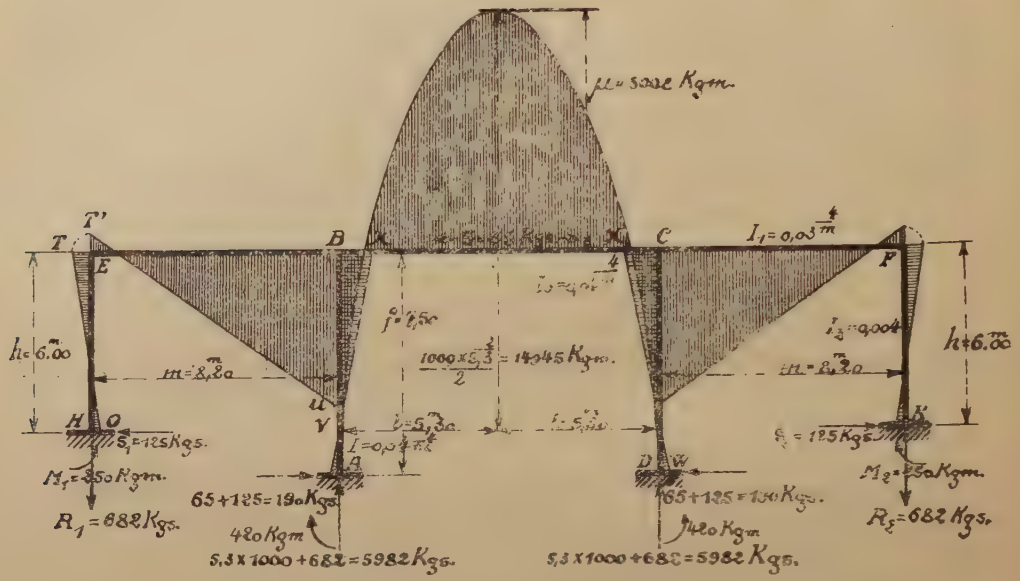

Fig. 20.

$$H_0 = 250\ \text{Kg.}\quad M = M_1$$

$$ET = ET' = 250 - 6 \times 125 = 500\ \text{Kg.-M.}$$

$$BU = -500 + 682 \times 8.2 = 5\,092\ \text{Kg.-M.}$$

$$BV = 14\,045 - 8\,008 = 6\,037\ \text{Kg.-M.}$$

$$6\,037 = 190 \times 7.5 + 5\,092 + DW$$

whence

$$DW = -1\,425 + 945 = -480\ \text{Kg.-M.}$$

$$BX = UV = -480 + 7.5 \times 190 = 945\ \text{Kg.-M.}$$

Note. — If in the same structure, we make $t = 2$ instead of $t = 10$, ft becomes 15 in place of 75 and

$$C_4 = 90\,[71\,(232 + 164 + 318) - 15 \times 60 \times 5.3] = 90\,(50\,694 - 4\,770) = 4\,133\,160$$

$$S = \frac{2 \times 1\,000 \times \overline{5.3}^{-3} \times 10.93}{4\,133\,160}(720 + 12 \times 11 - 562 \times 2) = \frac{56\,100 \times 740}{4\,133\,160} = 585\ \text{kgr.}$$

$$\alpha'_9 = \frac{2 \times 56.25 \times 6 \times 11 \times 71}{4\,133\,160} = 0.127$$

$$\alpha'_{10} = \frac{7.5 \times 6}{2 \times 4\,133\,160}[60(45 + 44) + 44(15 + 11)] = 0.0375.$$

$$\mu = \frac{2 \times 1\,000 \times \overline{5.3}^{2}}{6}(3 \times 0.1275 + 2 \times 5.3 \times 0.0375) = \frac{56\,180}{6} \times 0.781 = 7\,313 \text{ Kg.-M.}$$

$$S_1 = S_2 = -\frac{125 \times 14\,724}{5 \times 4\,133} = -89 \text{ kgr.}$$

$$M_1 = M_2 = 250 \times \frac{14\,724}{5 \times 4\,133} = 178 \text{ Kg.-M.} \quad R_1 = R_2 = -\frac{682 \times 14\,724}{5 \times 4\,135} = -486 \text{ kgr.}$$

These results show the considerable differences we get according as $/t = 10$ or $= 2$.

2. *Load of 1 000 kgr. per lineal metre uniformly distributed over each of the spans EB and CF.*

The formulæ to be applied are those numbered 1139 to 1147, except as to doubling the values of S, μ, $S_1 + S_2$, $M_1 + M_2$, $R_1 + R_2$ and to cancelling $S_1 - S_2$, $M_1 - M_2$, $R_1 - R_2$ and X.

$$S = \frac{2 \times 1\,000 \times \overline{8.2}^{3} \times \frac{4}{3}}{8 \times 14\,724\,000}[7.5(21.2 \times 10.93 + 75 \times 10.93 + 4 \times 75 \times 5.3) - 2 \times 5.3(12 \times$$

$$\times 10.93 + 3 \times 10 \times \overline{6}^{2} - \overline{7.5}^{2} \times 10)] = \frac{1\,473}{8 \times 14\,724}[7.5 \times 2\,641 -$$

$$- 10.6(132 + 1\,080 - 562)] = 160 \text{ kgr.}$$

$$\mu = -\frac{2 \times 1\,000 \times 8.2 \times \frac{4}{3} \times \overline{7.5}^{2} \times 10 \times 6}{24 \times 14\,724\,000}(3 \times 50 + 2 \times 10.93) = -$$

$$= -\frac{1\,472 \times 3\,375 \times 202}{24 \times 14\,724} = -2\,841 \text{ Kg.-M.}$$

$$S_1 = S_2 = \frac{S_1 + S_2}{2} = \frac{2 \times 1\,000 \times \overline{8.2}^{3} \times \frac{4}{3}}{2 \times 4 \times 14\,724\,000}[7.5 \times 2\,641 + 2 \times 5.3 \times 562] =$$

$$= \frac{1\,472 \times 25\,764}{14\,724 \times 8} = 322 \text{ kgr.}$$

$$M_1 - M_2 = \frac{M_1 + M_2}{2} = -\frac{2 \times 1\,000 \times \overline{8.2}^{2} \times \frac{4}{3} \times 7.5 \times 6}{2 \times 12 \times 14\,724\,000}(2\,641 + 2 \times 75 \times 5.3) = -$$

$$= -\frac{1\,472 \times 45 \times 3\,436}{24 \times 14\,724} = -644 \text{ Kg.-M.}$$

$$R_1 = R_2 = \frac{R_1 + R_2}{2} = \frac{2 \times 1\,000 \times 8.2 \times 7.5 \times 6}{2 \times 4 \times 14\,724\,000}[(3 \times 60 + 4 \times 10.93)\,2\,641 + 4 \times$$

$$\times 7.5 \times 5.3 \times 11] = 3\,816 \text{ kgr.}$$

The results just obtained make it possible to prepare, as shown below, the diagram (fig. 21) of the bending moments at each point of the structure.

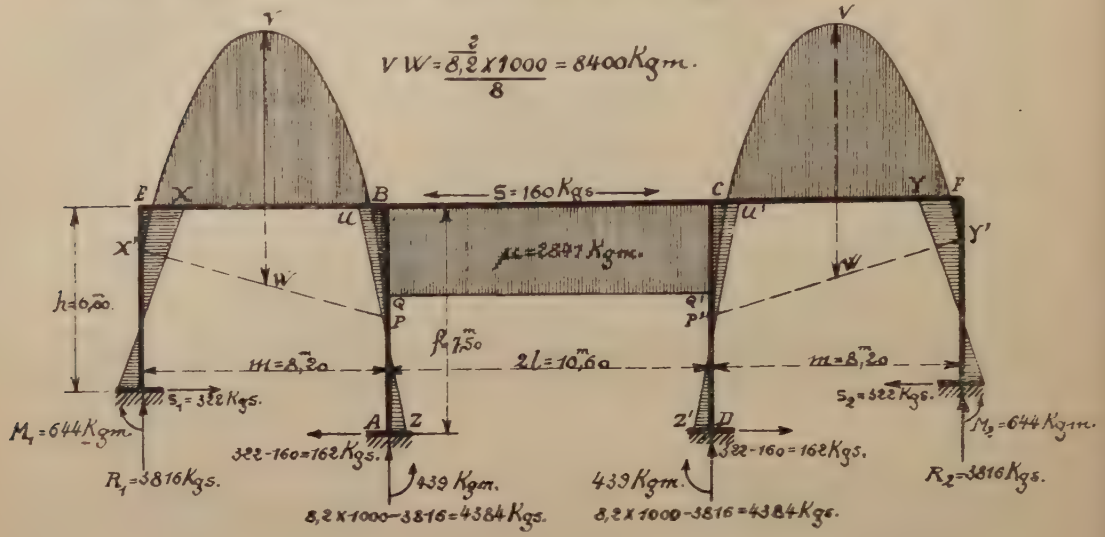

Fig. 21.

$$EX = EX' = FY = FY' = 322 \times 6 - 644 = 1\,288 \text{ Kg.-M.}$$

$$CP' = BC = 1\,288 + \frac{\overset{--2}{8.2} \times 1\,000}{2} - 3\,816 \times 8.2 = 3\,617 \text{ Kg.-M.}$$

$$AZ = DZ' = 7.5 \times 162 - 776 = 439 \text{ Kg.-M.}$$

$$BU = CU' = PQ = P'Q' = 3\,617 - 2\,841 = 776 \text{ Kg.-M.}$$

Note. — If for the same structure we have $t = 2$ instead of $t = 10$, ft becomes 15 instead of 75 and we have $C_4 = 4\,133\,160$ as found above at the 1. (note).

$$S = \frac{2 \times 1\,000 \times \overset{--2}{8.3} \times 11}{8 \times 4\,133\,160} [7.5\,(232 + 164 + 318) - 10.6\,(1\,080 + 132 - 112\,] = -$$

$$= -\frac{1\,472}{33\,065} \times 5\,335 = -281 \text{ kgr.}$$

$$y = -\frac{2 \times 1\,000 \times \overset{--2}{8\,3} \times 11 \times 2 \times \overset{--2}{7.5} \times 6}{24 \times 4\,133\,160} (3 \times 60 + 2 \times 11) = -$$

$$= -\frac{1\,472 \times 675 \times 202}{99\,195} = -2\,023 \text{ Kg.-M.}$$

$$S_1 = \frac{S_1 + S_2}{2} = \frac{2 \times 1\,000 \times \overset{--2}{8\,3} \times 11}{2 \times 4 \times 4\,133\,160} [7.5 \times 714 + 2 \times 5.3 \times 112] = +291 \text{ kgr.}$$

$$M_1 = \frac{M_1 + M_2}{2} = \frac{2 \times 1\,000 \times \overline{\overline{8.3}}^{2} \times 11}{2 \times 12 \times 4\,133\,160} \times 7.5 \times 6\,(232 + 164 + 477) = -582 \text{ Kg.-M.}$$

$$R_1 = \frac{R_1 + R_2}{2} = \frac{2 \times 1\,000 \times 8.2 \times 7.5 \times 6}{2 \times 4 \times 4\,133\,160}\,[224 \times 714 + 4 \times 15 \times 11 \times 5.3] = 3\,650 \text{ kgr.}$$

We see that the loadings differ considerably from the case in which $t = 10$.

3. *Horizontal thrust* $P_3 = 1\,000$ *kgr. acting towards the right at* B *due, for instance, to a vehicle having the brake applied.*

The formulæ 1040 to 1058 are to be applied in this case taking $b = f$ which gives

$$\mu = 0, \qquad S_1 + S_2 = 0, \qquad M_1 + M_2 = 0, \qquad R_1 + R_2 = 0$$

$$S = \frac{P_3}{2} = 500 \text{ kgr.}$$

$$X = \frac{3 P_3 f^3 l^2 m^2 l_1^2 h^2 t_2\,(2h^2 l_2 + 2hmt - f^3 t)}{8 N_1} =$$

$$= \frac{3 \times 1\,000 \times \overline{\overline{75}}^{2} \times 7.5 \times \overline{\overline{10.93}}^{2} \times 60 \times 6\,(2 \times 360 + 12 \times 10.93 - 75 \times 7.5)}{8 N_1} =$$

$$= \frac{3\,000 \times 1\,837\,665\,720 \times 289}{8 N_1}$$

N_1 is given by the formula 955 and equals :

$$N_1 = \frac{75 \times 5.3 \times 10.93}{4}\,\{(5.3 \times 10.93 + 3 \times 75 \times 10.93 + 4 \times 75 \times 5.3)\,[4 \times \overline{\overline{6}}^{3} \times 10\,(60 +$$

$$+ 10.93) + \overline{\overline{7.5}}^{3} \times 10\,(4 \times 60 + 10.93)] + 3 \times \overline{\overline{7.5}}^{2} \times 10 \times 5.3 \times 10.93\,(7.5 \times 10.93 +$$

$$+ 30 \times 60 + 4 \times 360) - 4 \times 75 \times 5.3 \times 60\,(\overline{\overline{6}}^{3} \times 10 + \overline{\overline{7.5}}^{3} \times 10)\,\} =$$

$$= \frac{4\,572}{4}\,[4\,123 \times 1\,672\,158 + 98\,380 \times 3\,322 - 95\,400 \times 6\,373] = 7\,226\,549 \times \overline{10}^{6}$$

$$X = \frac{1\,837\,665 \times 867}{8 \times 7\,226\,540} = 28 \text{ kgr.}$$

$$S_1 - S_2 = -\frac{P_3 f^3 t^2 l m t}{4 N_1}\,[f\,(4ht_2 + mt_1)\,(lmt_1 + 3flmt + 4ftl) + l\,(12fht_2 mt_1 + 3fm^2 t^2{}_1 +$$

$$+ 6h^2 t_2 mt_1 - 4f^2 tht_2)] = -\frac{1\,000 \times 42\,187 \times 58.3}{4 \times 7\,226\,549 \times \overline{10}^{6}}\,[7.5 \times 251 \times$$

$$\times 4\,123 + 5.3\,(720 \times 82.5 + 33 \times 82.5 + 2\,160 \times 11 - 4 \times 60 \times 562.5)] = -$$

$$= -\frac{2\,460 \times 7\,499\,160}{4 \times 7\,226\,549} = -638 \text{ kgr. whence } S_1 = -S_2 = -\frac{638}{2} = -319 \text{ kgr.}$$

$$M_1 - M_2 = \frac{P_3 f^3 t^2 lmt}{4N}\left[fh(2ht_2 + mt)(4lmt_1 + 3ftmt_1 + 4ftl) + 2h^2 t_2 l(hmt_1 - f^2 t)\right] =$$

$$= \frac{85.086}{\frac{6}{10}}\left[45 \times 131(4\,123 + 21.2 \times 11) + 10.6 \times 360(66 - 562)\right] =$$

$$= 85.086 \times 23.78 = 2\,022 \text{ Kg.-M.} \qquad M_1 = -M_2 = 1\,011 \text{ Kg.-M.}$$

$$R_1 - R_2 = -\frac{3P_3 f^3 t^2 lmt}{4mN_1}\left[fh^2 t_2(4lmt_1 + 3ftmt_1 + 2ftl) + 2h^3 t_2 l(ht_2 + mt_1)\right] =$$

$$= -\frac{3 \times 85.086}{8.2}\left[45 \times 60(4\,123 \times 21.2 \times 11 - 150 \times 5.3) + 10.6 \times 2\,160 \times 82\right] =$$

$$= -31.13 \times 11.492 = -358 \text{ kgr.} \qquad R_1 = -R_2 = -179 \text{ kgr.}$$

which enables the bending moment diagram below to be drawn (fig. 22).

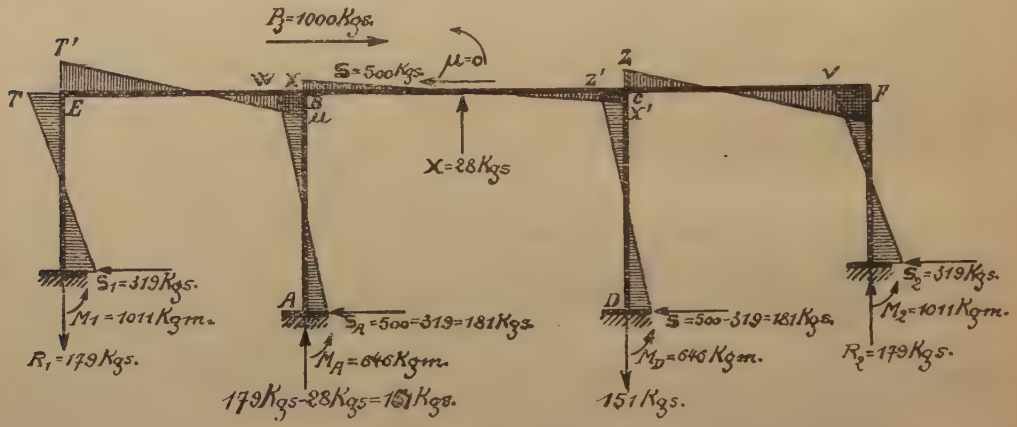

Fig. 22.

$$-FV = FV' = ET = ET' = 6 \times 319 - 1\,011 = 903 \text{ Kg.-M.}$$

$$-CZ = Bu = -179 \times 8.2 + 903 = -563 \text{ Kg.-M.}$$

$$M_A = 5.3 \times 28 + 563 - 181 \times 7.5 = -646 \text{ Kg.-M.}$$

$$-CZ' = BW = 181 \times 7.5 - 646 = 711 \text{ Kg.-M.}$$

$$-CX' = -BX = +Bu + BW = 711 - 563 = 5.3 \times 21 = 148 \text{ Kg.-M.}$$

Note. — It is easy to find the results in the case of $t = 2$ instead of $t = 10$.

4. *Contraction* — $2\lambda = 0.04$ m. *resulting in the section* BC *from a fall in temperature or from a shrinkage of the concrete.*

Formulæ to be applied : 1273 to 1282. Allowing $E = 2\,000\,000\,000$ for concrete I_0 being 0.04 we get :

$$S = -\frac{6 \times 2 \times \overline{10}^{9} \times 0.04 \times 0.02}{\overline{7.5}^{3} \times 10 \times 6^2 \times 10 \times 14\,724\,000}\{2\,641\,[\overline{7.5}^{3} \times 10(4 \times 60 + 10.93) + 4 \times \overline{6}^{3} \times$$

$$\times 10(60 + 10.93)] - 4 \times 5.3 \times 60\,[75(\overline{7.5}^{3} \times 10 + \overline{6}^{3} \times 10) + 3 \times 6 \times 10.93(60 \times 6 +$$

$$+ 6 \times 10.93 - \overline{7.5}^{2} \times 10)]\} = -\frac{96}{56.2 \times 36 \times 14\,724}[2\,641\,(1\,058\,969 + 613\,440) -$$

$$-1\,272\,(478\,425 - 26\,925)] = -\frac{96 \times 3\,842\,524\,169}{29\,816\,100} = -12\,372 \text{ kgr.}$$

For the piers jointed at the bases we get $S = -3\,084$ kgr. ; this difference is easy to understand in view of the additional resistance offered in the present case by the rigidity of the bases of the piers.

$$\mu = +\frac{96 \times 160 \times 11}{14\,724}[12(60 + 10.93) - \overline{7.5}^{2} \times 10] = \frac{105\,600 \times 289}{14\,724} = 2\,073 \text{ Kg.-M.}$$

$$X = 0, S_1 = S_2 = \frac{S_1 + S_2}{2} = -\frac{2 \times 96 \times 10}{2 \times 36 \times 14\,724}[7.5(4 \times 60 + 10.93)\,2\,641 - 2 \times$$

$$\times 5.3 \times 60\,(2 \times 562.5 - 3 \times 6 \times 10.93)] = -\frac{960\,(4\,971\,682 - 589\,572)}{530\,064} = -7\,946 \text{ kgr.}$$

$$M_1 = M_2 = \frac{M_1 + M_2}{2} = +\frac{2 \times 960 \times 6}{2 \times 530\,064}[7.5(2 \times 60 + 10.93)\,2\,641 - 2 \times$$

$$\times 5.3 \times 60\,(562.5 - 6 \times 10.93)] = +\frac{5\,760\,(2\,594\,782 - 315\,456)}{530\,064} = +24\,769 \text{ Kg.-M.}$$

$$R_1 - R_2 = \frac{R_1 + R_2}{2} = -\frac{6 \times 96 \times 100}{2 \times 8.2 \times 14\,724}[7.5 \times 2\,641 + 2 \times 5.3\,(360 + 2 \times 6 \times$$

$$\times 10.93 - 562.5)] = -\frac{57\,600\,(19\,807 - 742)}{241\,473} = -4\,548 \text{ kgr.}$$

This gives us the bending moment diagram below (fig. 23).

$Fu = ET = ET' = Fu' = 7\,946 \times 6 - 24\,769 = 22\,907$ Kg.-M.

$BV = CV' = 4\,548 \times 8.2 - 22\,907 = 14\,386$ Kg.-M.

$-2\,073 = -4\,426 \times 7.5 + 14\,386 - AW$

whence

$AW = DW' = -33\,195 + 16\,459 = -16\,736$ Kg.-M.

$BX = CX' = 16\,459$ Kg.-M.

It will be noticed that the moments are much higher than those found in the case

of the jointed piers : from this point of view of expansion and contraction of BC, the piers jointed at the bases are better than those rigidly held.

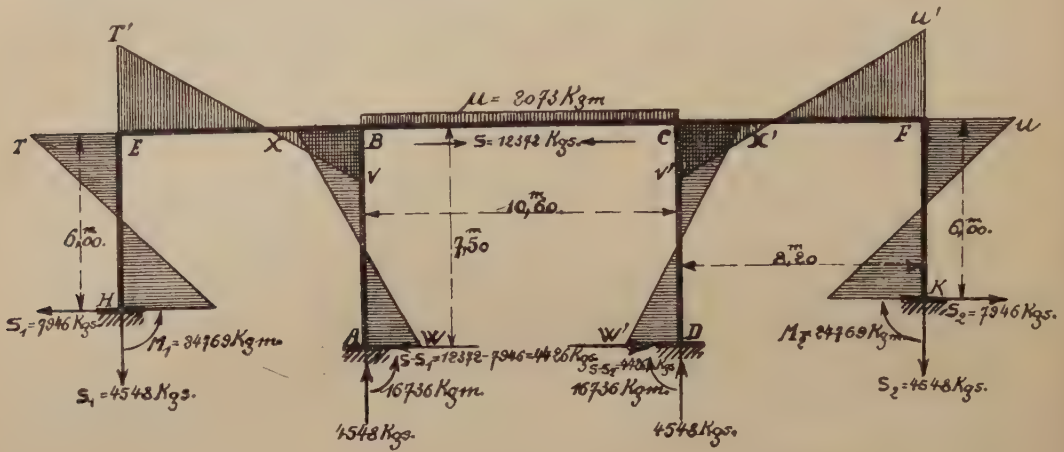

Fig. 23.

NOTE. — If for the same structure we made $t = 2$ instead of $t = 10$, which would give $\int t = 15$ instead of 75, we should get $C_1 = 4\ 133\ 160$ as before and obtain results very different from those found above for S, μ, S_1, M_1, R_1.

5. *Contractions* $e = 0.03$ *m. and* $e' = 0.03$ *m. respectively of the sections* BE *and* CF.

Taking into account the last note on the ninth case of loading considered above, it is proper to apply the formulæ 1222 to 1232 changing therein the sign of e' in the case of contraction and of e in the case of expansion and making $\Delta_{2A} = 0$, which gives when $E = 2 \times \overline{10}^{9}$:

$$S = -\frac{2 \times 0.03 \times 3 \times 2 \times \overline{10}^{9} \times 0.04 \times \overline{7.5}^{2} \times 10}{\overline{7.5}^{2} \times 10 \times 6 \times \overline{10}^{2} \times 14\ 724\ 000}(4\ 971\ 682 - 589\ 572) =$$

$$= -\frac{144 \times 10 \times 4\ 381\ 110}{530\ 064} = -11\ 919 \text{ kgr}$$

$$\mu = -\frac{2 \times 0.03 \times 3 \times 2 \times \overline{10}^{9} \times 10.93 \times 562 \times 0.04}{14\ 724\ 000} = -\frac{144 \times 100 \times 6\ 182}{14\ 724} = -6\ 046 \text{ Kg.-M.}$$

$$S_1 = S_2 = \frac{S_1 + S_2}{2} = -\frac{2 \times 0.03 \times 6 \times 2 \times \overline{10}^{9} \times 0.04}{2 \times 360 \times 14\ 724\ 000}(4\ 971\ 682 - 4 \times 5.3 \times$$

$$\times 562.5 \times 60) = -\frac{1\ 440 \times 4\ 256\ 182}{530\ 064} = -11\ 564 \text{ kgr.}$$

$$M_1 = M_2 = \frac{M_1 + M_2}{2} = + \frac{2 \times 0.03 \times 6 \times 2 \times 10^{-9} \times 0.04 \times 6}{2 \times 360 \times 14\,724\,000} (2\,594\,782 - 2 \times 5.3 \times$$

$$\times 562.5 \times 60) = + \frac{8\,640 \times 2\,237\,032}{530\,064} = 36\,468 \text{ Kg.-M.}$$

$$R_1 = R_2 = \frac{R_1 + R_2}{2} = - \frac{2 \times 0.03 \times 18 \times 2 \times 10^{-9} \times 0.04 \times 7.5}{2 \times 8.2 \times 14\,724\,000} (2\,641 - 2 \times$$

$$\times 5.3 \times 75) = - \frac{8\,640 \times 10 \times 1\,846}{241\,473} = - 4\,957 \text{ kgr.}$$

This gives us the bending moment diagram (fig. 24).

$$Fu = Fu' = ET = ET' = 11\,564 \times 6 - 36\,468 = 32\,916 \text{ Kg.-M.}$$

$$BV = CV' = - 32\,916 + 4\,957 \times 8.2 = + 7\,731 \text{ Kg.-M.}$$

$$6\,046 = - 355 \times 7.5 + 7\,731 - AW$$

whence : $AW = DW' = - 6\,046 - 2\,662.5 + 7\,731 = 937$ Kg.-M.

$$BX = CX' = 7\,731 - 6\,046 = 1\,685 \text{ Kg.-M.}$$

We see by these results that the moments in the columns EH and FK are very high at the ends in the case of loading considered.

In the case in which the piers are hinged at the base instead of being rigidly held, we get from formulæ obtained in a previons study under the Nos. 906, 907, 908,

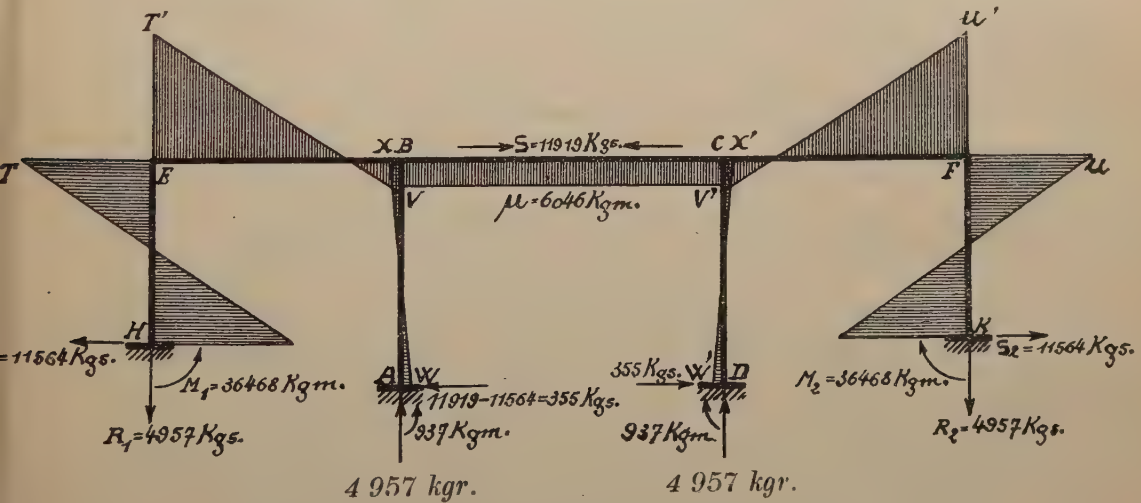

Fig. 24.

and 909, completed by 714, 715, 726 and 727, S = — 2 952 kgr.

$\mu = - 3\,216$ Kg -M., $S_1 = S_2 = - 2\,855$ kgr. $R_1 = R_2 = - 1\,582$ kgr.,

or a moment, at E of : $6 \times 2\,855 = -7\,130$ Kg.-M. instead of 32 916 Kg.-M. as with the columns rigidly held and 0 at H instead of 36 468 Kg.-M.

NOTE. — If for the same structure we take $t = 2$ instead of $t = 10$, which gives $fl = 15$ instead of 75, we get $C_1 = 4\,133\,160$ as found previously and finally we arrive at results very different from those obtained above.

6. *Resistance of the ground $p_7 = 100$ kgr. uniformly distributed over the whole length of each of the uprights* EH *and* FK.

The formulæ to be applied are those numbered 1203 to 1211, except for doubling the values of S, μ, $S_1 + S_2$, $M_1 + M_2$, $R_1 + R_2$ and for making $X = 0$, $S_1 - S_2 = 0$, $M_1 - M_2 = 0$, $R_1 - R_2 = 0$ which gives :

$$S = \frac{2p_7h^2}{8C}\left[f(4lmt + ftmt_1 + 4ftl)(4ht_2 + 3mt_1) + 2ht_2l(hmt_1 - 2f^2l)\right] =$$

$$= \frac{200 \times 36}{8 \times 14\,724\,000}\left[7.5(233 + 825 + 1\,590)\,273 + 636(66 - 1\,125)\right] =$$

$$= \frac{900}{14\,724\,000}\left[7.5 \times 2\,648 \times 273 - 636 \times 1\,089\right] = 289 \text{ kgr.}$$

$$\mu = \frac{2p_7h^4t_2f^2tmt_1}{24C_1} = \frac{200 \times 12\,960 \times 562 \times 11}{24 \times 14\,724\,000} = 45 \text{ Kg.-M.}$$

$$S_1 + S_2 = -\frac{2p_7fh^2}{4C_1}\left[(4lmt + ftmt + 4ftl)(4ht_2 + 3mt_1) - 4fthlt_2l\right] = -$$

$$-\frac{200 \times 7.5 \times 36}{4 \times 14\,724\,000}\left[2\,648 \times 295 - 300 \times 60 \times 5.3\right) = -650 \text{kgr.}\; S_1 = S_2 = -\frac{650}{2} = -325 \text{kgr.}$$

$$M_1 + M_2 = \frac{2p_7fh^3}{12C_1}\left[(4lmt + ftmt + 4ftl)(2ht_2 + 3mt_1) - 2fthlt_2l\right] =$$

$$= \frac{200 \times 7.5 \times 216 \times (405\,144 - 47\,700)}{12 \times 14\,724\,000} = 722 \text{ Kg.-M. } M_1 = M_2 = 361 \text{ Kg.-M.}$$

$$R_1 + R_2 = \frac{2p_7fh^4t_2}{4mC_1}(4lmt + ftmt + 2ftl) = \frac{200 \times 7.5 \times 12\,960 \times (2\,648 - 795}{4 \times 8.2 \times 14\,724\,000} = 74 \text{ kgr.}$$

$$R_1 = R_2 = \frac{74}{2} = 37 \text{ kgr.}$$

These results enable us to draw the diagram below of the bending moments (fig. 25).

The horizontal force at A and D equals $600 - 325 - 289 = -14$ kgr.

The moment at E is : $EW = +325 \times 6 - 361 - \frac{100 \times 6^2}{2} = -211$ Kg.-M.

$BV = CV' = -211 + 37 \times 8\,3 = +96$ Kg.-M.

$M_A = +96 - (14 \times 7.5 + 45) = -66$ Kg.-M.

$CX' = BX = +14 \times 7.5 - 66 = -39$ Kg.-M.

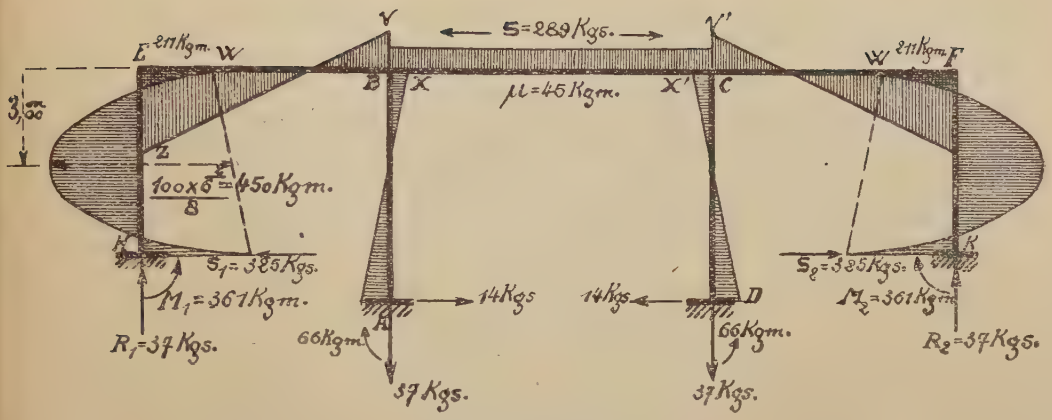

Fig. 25.

NOTE. — We can readily find the values of S, μ, $S_1 + S_2$, $M_1 + M_2$, $R_1 + R_2$ for the case of $t = \frac{I_0}{I} = 2$ instead of 5.

We shall have for μ for example :

$$\frac{200 \times 12\,960 \times 112 \times 11}{24 \times 4\,135\,160} = 33 \text{ Kg.-M.}$$

The co-ordination of the duties of the Operating and Locomotive Running Departments and their organic reunion,[1]

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Figs. 1 to 3, pp. 46 to 49.

INTRODUCTION.

The intense and rapid growth of passenger and goods traffic after the war forced the great railway companies to carry out quite a series of enquiries on the scientific reorganisation of their various departments, and in particular, of those directly concerned in the movement of traffic.

The Operating Department which decides the running of the trains, and the Locomotive Running Department which is responsible for the movements of the locomotives, have felt more than the other departments the pressing need for reorganisation.

The closer collaboration of these two branches of the service assuring the movement of the trains has resulted in organic reform on different railway administrations.

It would be of interest to study the existing state of affairs on the great railways as regards the useful co-ordination of the duties of these two departments both at headquarters and outside. The result will be to show that in consequence of local conditions the liaison of the Operating and Running Departments is assured in a distinct way,

— either by the *general co-ordination of duties of the Headquarters Services,*

— or by *close local co-ordination of the Outdoors Sections,*

as follows :

1. — of the area controls of the Operating and Running departments in the divisions of the system;

2. — of the area traffic regulating centres (Dispatching-System, Special Train working staff, Train Control System);

(1) Translated from the French.

3. — of the stations and the depots.

The character of the traffic, the more or less variable manner in which it collects, the legislative bases of the general operation and the particular bases of the regulations governing the movement of the trains and of the work done by the staff, as well as the more or less highly developed administrative centralisation on different systems, exercise considerable influence upon the form in which this co-operation between the services mentioned expresses itself.

I

Study of the co-ordination of the duties of the Operating and Locomotive Running Departments on the Paris, Lyons and Mediterranean Railway.

The Paris, Lyons and Mediterranean Company operates a railway of a length of 9 821 km. (6 103 miles). This system elongated in form (fig. 1) is almost systematically divided into two parts, by the vertical axis formed by the main line from Paris to Marseilles through Lyons.

The districts served are for the most part agricultural, horticultural, or viticultural, the exceptions being the ore fields of the Loire, and the industrial areas about Lyons and Grenoble, the steady growth of which have their influence on the increase of traffic.

The intense passenger traffic on this system is seasonal towards its Southern part — the Riviera — in winter, and towards its Eastern part — the two Savoys — in summer.

The goods traffic is regular, but the traffic currents are quite different as regards their speed.

G. V. — (Grande Vitesse). — The agricultural products exported, such as vegetables, fruit and flowers, demand fast services and make it necessary to organise services of goods trains running at high speeds, known as express goods trains.

This traffic is unilateral in origin :

— from the port of Marseilles and the producing districts of the South, such as :

  — the coastal districts of the Mediterranean;
  — the Rhone valley;
  — the Vienne district and that South of Lyons,

and flows toward the North, the destinations being :

— Paris;
— the ports of exportation to Great Britain;
— Belgium;
— Germany;
— Switzerland.

P. V. — (Petite Vitesse). — The ordinary goods traffic originating in the South generally does not leave France. This traffic, directed from the South to the North, quickly loses its density and, checked at Lyons, is to a large extent replaced by traffic in the opposite direction coming from the East and from the

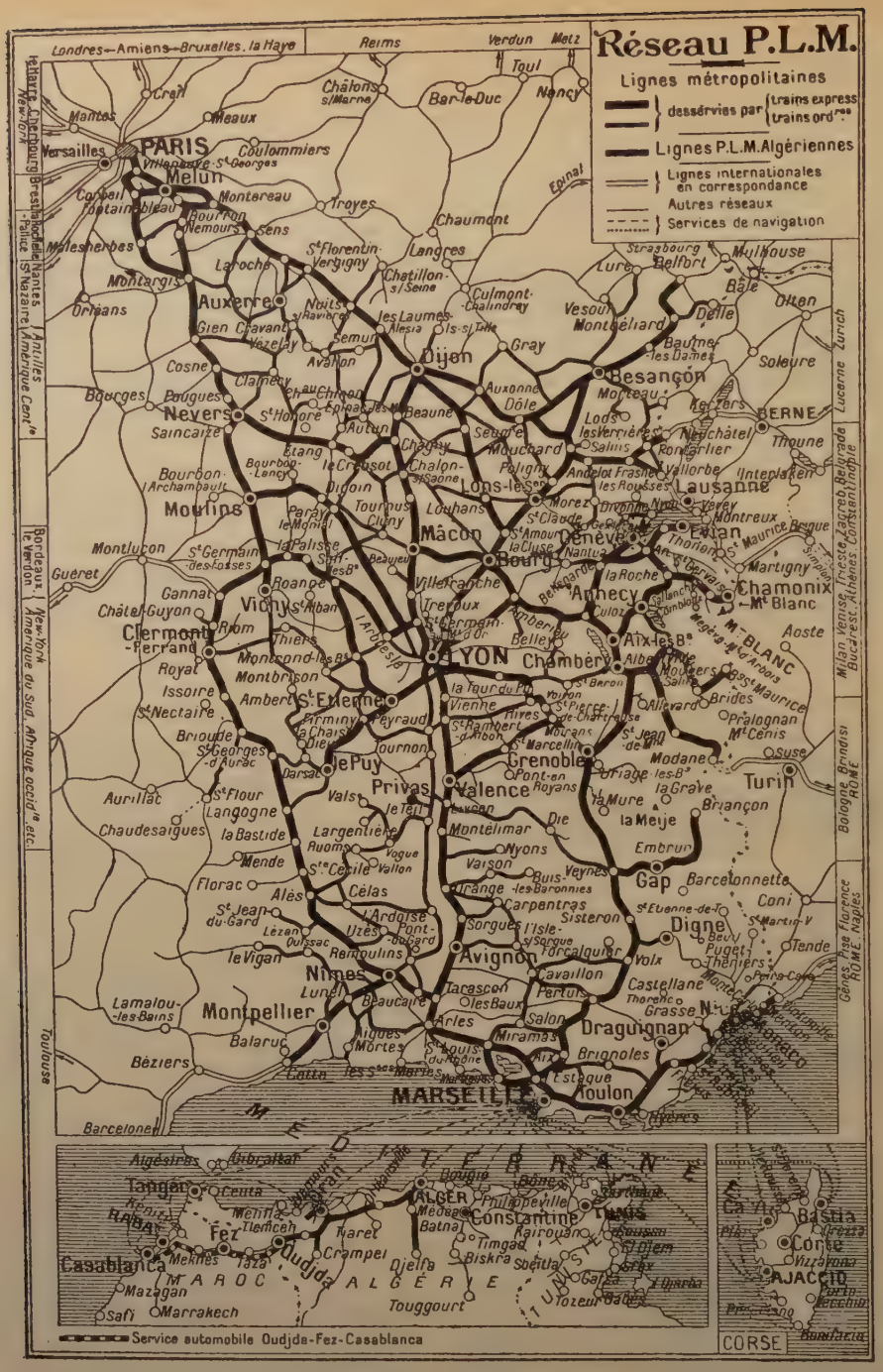


Fig. 1. — Map of the Paris, Lyons and Mediterranean Railway.

Explanation of legend (right top corner):

P. L. M. (Paris, Lyons and Mediterranean) System.

Lines in France over which { express trains / ordinary trains } run.

P. L. M. lines in Algeria.

International lines in connection with the P. L. M.

Other railways.

Steamship services.

North, which comes on to the Paris, Lyons and Mediterranean System, in particular at :

— Is-sur-Tille, coal and iron from the East and North of France, the Saar and the Ruhr;

— Villeneuve - Saint - Georges, goods from the Paris area, coal from the North, and important goods through the Channel and Atlantic ports.

The large number of express passenger trains in both directions, and the very great number of express goods trains, in connection with which punctual delivery is very important, has resulted in the very marked *parallelism* (fig. 2) of the *train working diagram* in order to be able to reap the *advantages of the rational organisation of the work on* more or less *homogeneous mass production lines.*

* * *

As regards organisation, the system is divided into :

11 Operating Divisions, each under an Operating Superintendent who represents in the Section the Chief Engineer of the Operating Department stationed at Paris;

8 Running Divisions, controlled by Engineers in the Running Department responsible for the Division to the Chief Engineer of the Running Department, or the Chief Engineer of the Rolling Stock and Running Department at Paris;

3 Rolling Stock Divisions, each under an Engineer Controlling the principal Workshops under the Chief Engineer of the Rolling Stock Department or the Chief Engineer of the Rolling Stock and Running Department at Paris;

12 Permanent Way Divisions, controlled by Engineers under the Chief Permanent Way Engineer at Paris.

The Divisional Headquarters :

— Operating,
— Locomotive Running,
— Rolling Stock,
— Permanent Way,

are stationed at the following centres :

Operating.	*Locomotive running.*	*Rolling stock.*	*Permanent way.*
Paris.	Paris.	Villeneuve-St-Georges.	Paris I/II.
Nevers.	Nevers.	—	Nevers.
Dijon.	Dijon.	—	Dijon.
Lyons.	Lyons.	Oullins.	Lyons I/II.
Clermont-Ferrand.	—	—	Clermont-Ferrand.
Saint-Etienne.	Saint-Etienne.	—	—
Valence.	—	—	Valence.
Marseilles.	Marseilles.	Arles.	Marseilles.
Nimes.	Nimes.	—	Nimes.
Grenoble.	—	—	Grenoble.
Chambéry.	Chambéry.	—	Chambéry.

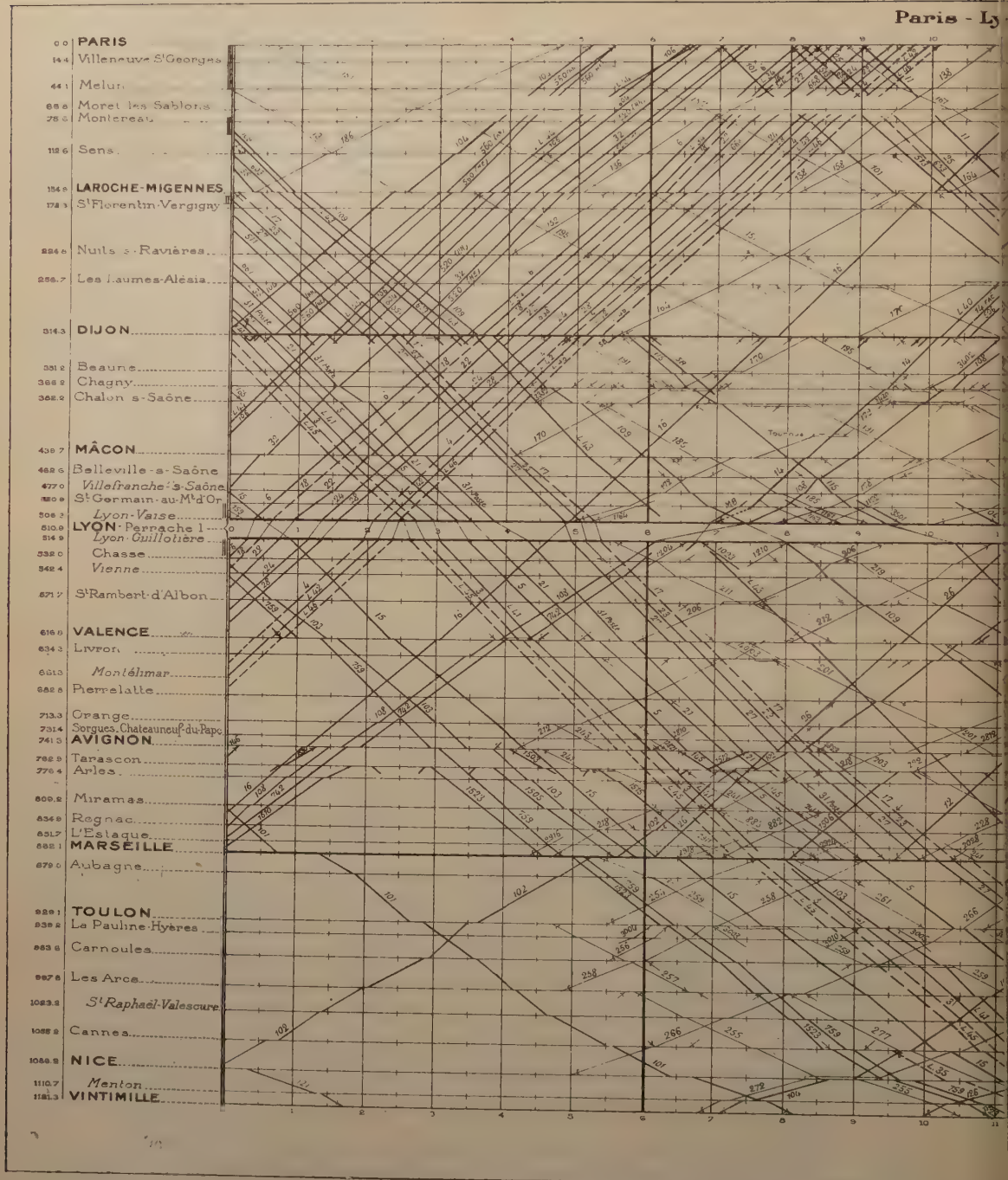
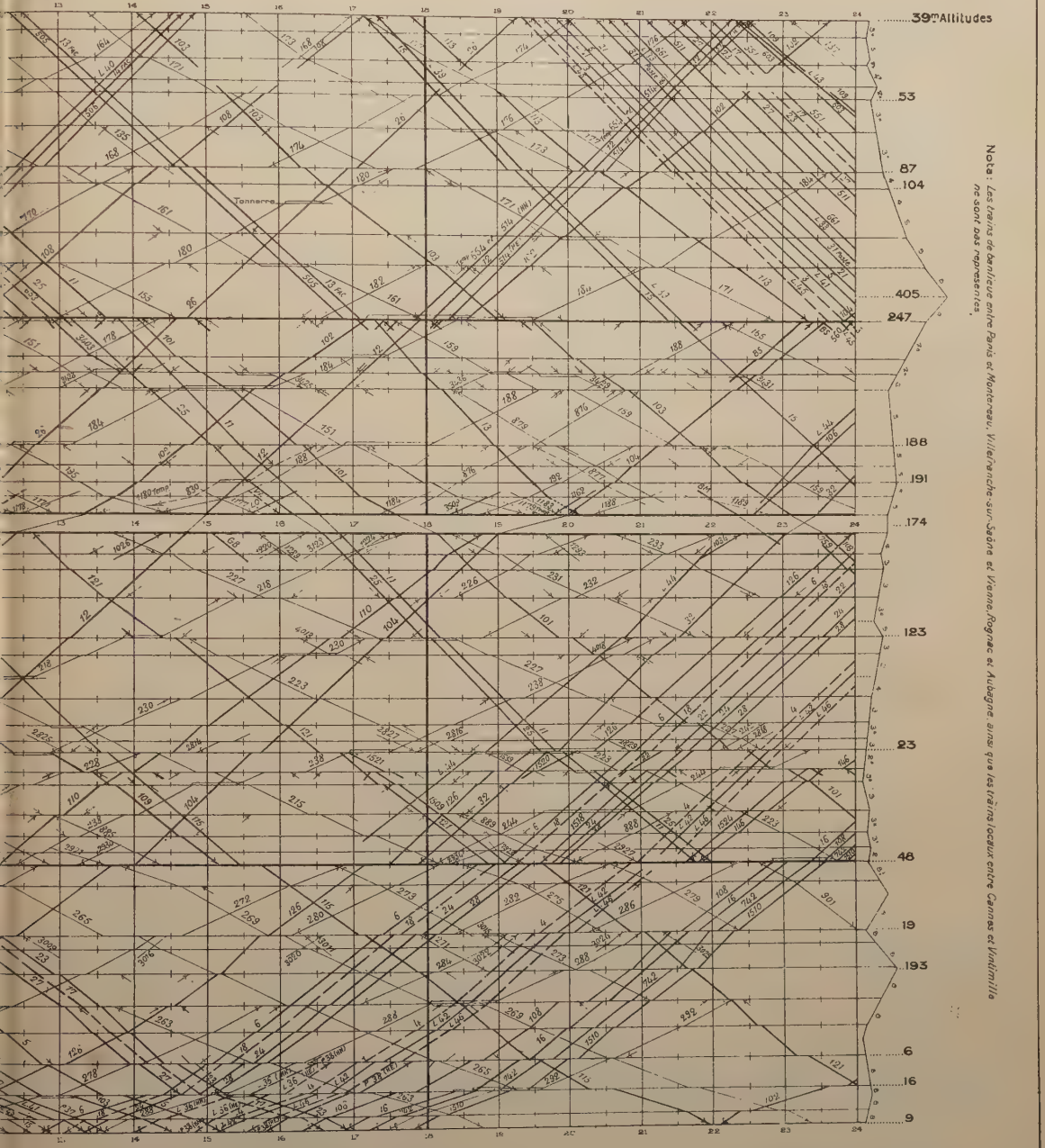


Fig. 2. — Graphical time-table of passenger trains (exp

N. B. — The suburban trains between Paris and Montereau Villefranche-sur-Saône and Vi



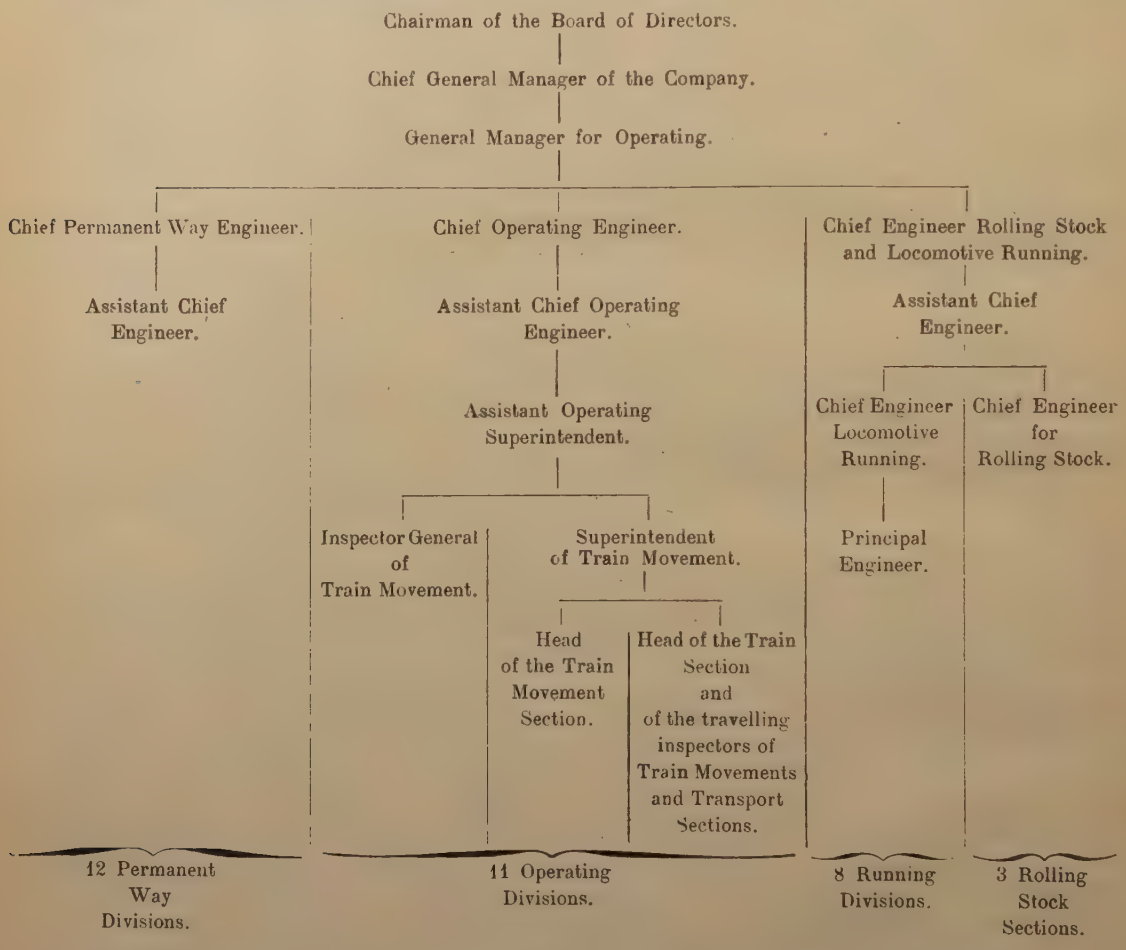
st and stopping) on the main line Paris to Vintimille.

nac and Aubagne, as also the local trains between Cannes and Vintimille, are not shewn.

The divisions of the Operating, Locomotive Running and Permanent Way Departments do not quite coincide, their boundaries sometimes overlapping.

* * *

The Headquarters' services of the Company are located at its Head Office in Paris. The general organisation is shewn diagrammatically below :


Chairman of the Board of Directors.
Chief General Manager of the Company.
General Manager for Operating.
Chief Permanent Way Engineer.
Assistant Chief Engineer.
Chief Operating Engineer.
Assistant Chief Operating Engineer.
Assistant Operating Superintendent.
Inspector General of Train Movement.
Superintendent of Train Movement.
Head of the Train Movement Section.
Head of the Train Section and of the travelling inspectors of Train Movements and Transport Sections.
Chief Engineer Rolling Stock and Locomotive Running.
Assistant Chief Engineer.
Chief Engineer Locomotive Running.
Principal Engineer.
Chief Engineer for Rolling Stock.
12 Permanent Way Divisions.
11 Operating Divisions.
8 Running Divisions.
3 Rolling Stock Sections.

The analysis into the co-ordination of the duties and of the joint management of the two services :

— Train working,

— Locomotive running,

brings out the following degrees in which they can usefully co-operate :

I. — **At Headquarters**.

The Director of Operating having been made responsible for the general co-ordi-

nation of the management of the three departments by delegation from the Chief General Manager, arbitrarily assures their liaison and the necessary collaboration.

The Chief Engineers of the

— Operating,
— Rolling Stock and Locomotive Running,
— Permanent Way Departments,

correspond

— either *personally*
— or by *administrative channels*

when they have questions common to all to settle.

— The meetings of the Engineers-in-Chief presided over by the

— Chief General Manager or by the
— Manager of Operating Services

deal with all general questions affecting the satisfactory working of the different departments.

— By authority from the Chief Engineers of the

— Operating Department,
— Rolling Stock and Locomotive Running Department,

their immediate Assistants, namely :

— the Assistant Chief Operating Engineer, who may be represented by the Superintendent of Train Movements, and
— the Chief Engineer for Locomotive Running, who may be represented by the Principal Engineer of the Locomotive Running Department,

come to agreement on questions concerning :

— the train times,
— the loads the engines can work and the distribution of the locomotives over the system.

On the other hand, the turns of duty of the enginemen are quite independent of those of the train staff.

The train times and speeds are regulated by the Operating Department, always of course taking into account the capabilities of the different types of locomotives in use on the system.

The timings of the important trains are definitely settled at the Joint Conference of the Train Service Inspectors held at the beginning of each year at the Operating Headquarters at Paris. The timing of local trains and goods trains is in most cases arranged by the Train Inspectors under the Principal Inspectors at the Divisional Headquarters.

The turns of duty of the train staff are decentralised in the Operating divisions, as are those of the locomotive men, which later are fixed by the Divisional Engineer of the Running Department in each section in agreement with the heads of the adjoining sections.

Under the Chairmanship of the General Operating Manager, a direct liaison has been established between

— the Superintendent of Train Movement,
— the Principal Engineer, Running Department,

both of whom are responsible in turn for ensuring the closest understanding possible between the two working services in question.

The tours of inspection made by the Inspector General of Train Movement enable him to intervene directly in order to remove all temporary difficulties in the working of the Operating and Locomotive running services, and which could be settled on the spot by the immediate

and active assistance of any one of the services concerned :

— by alterations in the working of certain trains;

— by drawing up new paths for light engines in order to return them quickly :

— to the starting point;
to marshalling yards;

— by alterations in the load of goods trains according to the capacity of the locomotives available.

II. — **Divisional Headquarters.**

— The Principal Inspectors, Heads of an Operating Division,
— The Engineers of the Locomotive Running Department, Heads of a Locomotive Running Division,
— The Divisional Chief Engineer of the Permanent Way Department

only confer together at irregular intervals, but *co-operate in all particular cases* which occur in which two or three divisional services, as mentioned above, are affected, as for example :

— reduction of the regular speed of trains due to relaying, etc., sleepering, ballasting, renovating the track or bridges, etc., when daily orders and service instructions are signed by the Divisional Heads of the three services affected.

The Divisional Heads of the Departments also discuss :

— the notice to be given for locomotives required for « as required » and « special » trains,
— the timings to be regularised for the working of light engines,
— the times at which the rakes of carriages will be made available for washing.

As regards repairs to engines in the Sheds, engine workings, as well as movements of enginemen, on the one hand, and on the other — the preparation of workings for local goods and passenger trains, as well as movements of train staff — the two departments are completely independent.

The *Operating Department* in principle *orders* « as required » and « special » trains to be run, which the *Locomotive Running Department ought to be able to cover* readily whether they be regular or not.

The Locomotive Running Department in return takes the initiative in :

— asking for the re-timing of a booked train which does not fit in well with the movement of the engines or of the Locomotive Department Staff,

— preventing the running of an additional train :

— the path of which is bad,
— the load of which is too small to justify it,

— following very closely the loading of goods trains and reporting to the Operating Department those not made up to full load. In this case, the Operating Department takes steps to put the matter right.

II*a* — **Dispatching-System.**

The regulation of the traffic on the Paris, Lyons and Mediterranean System

is done by three distinct organisations, namely :

1. The Central Train Dispatching Offices;
2. The Train Control of the Lyons area;
3. The Offices of the Controllers of Train Movement in stations.

The map (fig. 3) shews the present position of these organisations and their respective areas at the beginning of 1928.

1. *The role of the Dispatching-System Central Offices.*

The *Dispatching-System* adopted by the Paris, Lyons and Mediterranean system consists of the *concentration* of all *information* regarding the *movements* of trains in the hands of a single employee known as the « Dispatcher » and connected by a special telephone system to the :

— *stations,*
— *signal boxes;*
  — of a line or
  — of a group of lines,
    — either *double track*
    — or *single track.*

The *working* of this organisation, and the conditions under which the Dispatcher gives orders in the stations and the signal boxes, *derogates* to some extent from certain *requirements* of the *General Operating Regulations* in force on the Paris, Lyons and Mediterranean system from the fact of the liaison which is established between :

— the *Dispatcher,*
— the *stations* and *signal boxes*

of the line or group of lines on which the Train Dispatching System has been put into operation.

The essential *role* of the Dispatcher is to :

— *supervise,*
— *guide*

the *movement* of the trains on the line or the group of lines in the zone he is controlling.

In principle, the Dispatcher *does not interfere* in the *station working.*

His *function* as regards the stations and signal boxes is to :

— *supply* them with *information* affecting the *movement* of the trains,
— *advise* them as to the correct *steps* to be taken to :
  — *eliminate* the causes of *delay,*
  — *prevent blocking* the :
    — *stations,*
    — *sections* of the line,
— *prescribe,* in certain *determined cases,* the *steps* the situation of the *movement of the trains as a whole demands.*

The map (fig. 3) shews clearly the lines of the Paris, Lyons and Mediterranean system on which the Dispatching-System has been in operation since 1928.

The main lines controlled by the Dispatching-System are divided into consecutive circuits, controlled by the respective Central Offices.

Local arrangements decide the conditions under which the Dispatchers of two adjacent divisions transmit to one another the information to be forwarded from one section to another or inversely.

The Dispatcher is kept constantly informed as to the actual state of the traffic as a whole.

The stations and signal boxes of the line, to which he is connected by special

Fig. 3. — Map of the lines on the Paris, Lyons and Mediterranean System equipped with Dispatching System instruments.

Explanation of legend (*right top corner*) :

Dispatching-System Circuits and Dispatchers Offices.

Lines equipped.

Lines being fitted.

DIJON — Central Dispatching Office (2 circuits) and Office of Head of train movements in course of realisation.

PERRIGNY — Office of the Head of train movements.

NEVERS — Central Dispatching Office (1 circuit).

telephone, advise the Dispatcher at frequent intervals of any matter concerning the running of trains.

They advise him in particular as to :

1. The *actual times* :
   — of arrival,
   — of departure,
   — of passing

of all trains;

2. The *delays* in the normal running of the trains;

3. Particulars concerning the *loading* of the trains :
   — their *formation*,
   — their *sorting out*;

4. Advices of *starting* :
   — *supplementary* trains,
   — *as required* trains;
   — *special* trains,
   — *isolated* engines;

5. Advices of the *cancellation* of :
   — *regular* or other trains,
   — *light* engines;

6. *Alterations* made in the normal order in which the trains *follow one another* :
   — *shunting into holding sidings*,
   — *dispatching out of turn*;

7. Advices of exceptional *work* or *shunting* :
   — requests to stop at certain stations,
   — attaching wagons which may delay certain trains at their station;

8. Various *incidents in the movement of the trains* that may affect regularity of running :
   — failure of block apparatus or signals,
   — failure of couplings,
   — failure of engines,
   — requests for assistance,
   — derailments,
   — defective lines,
   — using avoiding lines,
   — double heading, etc.;

9. Advices on the *instruments of the Dispatching-System* :
   — their defective working,
   — their failure;

10. *Messages on the distribution of rolling stock*,
   — as necessary.

The stations and signal boxes maintain the register of all communications transmitted to the Dispatching System Offices.

By the help of the indications supplied them in this way on the running of the trains, the Dispatching Offices usefully fulfil their function of :
   — *counsellors* for the *Operating* Department,
   — *intelligence agents* for the *Locomotive* Department.

The Dispatchers *advise automatically* the stations concerned to which they are connected by telephone of :
   — the *delays* to the trains in order to assist :
      — their *dispatch out of turn*,
      — their *running forward late*;
   — the *position* of the trains running closey together in front of :
      — a faster train;
   — the available holding sidings, etc.

In return, the Head of Service in the stations ought to freely resort to the :
   — information given by,
   — advice of

the Dispatcher as he requires it, particularly in periods of

— intense,
— irregular

traffic movements, especially in the event of :

— *congestion* of certain stations,
— *interruption* of any lines,
— *using avoiding lines.*

2. *The functions of the Control Office for the Lyons Area.*

The Area Control Office common to both the :

— Operating,
— Locomotive Running, Departments

is at the Divisional Operating Headquarters at Lyons, Lyons being the station at which *engines are generally changed,* and nearly all *goods trains* are *shunted* and *re-formed;* such *co-operation* of the men of the two services controlling the proper flow of the traffic has been *particularly necessary and useful.*

The Lyons Control Office has as its *objects :*

— the *co-ordination* of the *work effected* by the *principal stations* in the Lyons area,
— the *closer understanding* and *co-operation* of the *Operating* and *Locomotive Running Departments.*

It is fitted out with *many telephone lines* which converge at the office and enable it to communicate quickly with everyone concerned.

It can also get into touch with the stations by the Dispatching-System telephones installed in the Lyons Central Dispatching Office which is close to, and is under, the Area Control Office.

In this Common Control Office are stationed :

— Two Inspectors of the *Operating Department,*
— Two Inspectors of the *Locomotive Department,*

who are responsible for the *supervision of the movement of trains around Lyons at all times.*

The functions of these higher grade men, to whom has been given the fullest liberty of action, and who act in their sections as the delegates of their respective Divisional Chiefs, consist in :

— actively *supervising* and *controlling* without delay the general state of the *traffic,*
— *improving the work in the stations and co-ordinating the operations made therein.*

In order to *supervise* and *control* the train movements, the Control Office has always available *graphical time-tables* on which the Dispatchers *materialise* the exact situation of the *actual running of the trains.*

The Control Office is thereby kept *au courant* of every *operation,* is aware of the least *irregularity,* and can obtain information about the smallest *incident* from the stations and signal boxes in the *shortest possible times.*

It very closely *follows* all *variations in the traffic, proposes* all the *steps* required *to meet them* without delay and in the most economical way, regulates the *inwards flow of traffic* at the large stations having *fans of holding sidings,* and by a *scientific use* of these fans, as well as *by making use* of the *goods trains workings,* exerts a large influence on the proper *turn round of the rolling stock* and thereby *improves* the *co-efficient of use.*

On the other hand, the stations are

given accurate information by the Control Office on the position of the trains they expect to receive in such a way that *holding the trains at signals* can be reduced to a minimum.

The *holding of trains in sidings* and *forwarding them out of turn* are regulated in consequence with full knowledge of the reasons, which very greatly helps the *output* of the :

— *marshalling yards,*
— *sections of the line,*

in the event of their accidentally becoming congested, and in turn makes easier :

— the *working* of trains over *alternative lines,*
— the *distribution of the staff* between the Control Offices.

On the maintenance of a *regular movement* depend, in fact, the *improvement of the working of the stations,* and the *co-ordination of the operations* carried out therein. But these two things which there appear to be the final object, are even the condition to be observed if the time-tables are to be followed and a good efficiency obtained from

— the *plant,*
— the *staff.*

For this reason the Control Office, in agreement with the Train Movement Office of the Divisional Operating Office at Lyons, is given charge also of the following matters :

— making full use of the loading of the trains,
— the proper running of regular or semi-regular trains,
— ordering « as required » trains,
— use of the train staff and enginemen,
— running of pick-up goods trains,
— distribution of rolling stock, etc.

The Inspectors :

— of the Operating Department,
— of the Locomotive Running Department,

attached to the Lyons Control Office *work together* as regards *the running of supplementary trains* by taking into account:

— the *requirements of the stations,*
— the *capabilities of the depots.*

The control Office is not in this case a link between the stations and the depots which ought to maintain :

— *direct contact*

but should simply help to form a *closer* and *more intimate contact* between them both, in order to obtain :

— *punctual running of the trains,*
— *economic use* of the available *engines.*

With this object in view, the Locomotive Running Inspector, in agreement with his colleagues on the Operating side:

— supervises the use made of the locomotives,
— takes the necessary steps to prevent the engines
— leaving,
— arriving,

from standing needlessly anywhere.

In addition, he occupies himself as to *the regularity with which the engines are supplied to work :*

— booked,
— « as required » goods trains,

started from the stations in the Lyons area.

He co-ordinates the capabilities of the Depots in this area in order that engines for working the trains may be available regularly, at all costs and to time.

This action of the Control Office ne-

cessitates the stations and depots submitting to a *levelling* of their respective *needs* and *capacities*, to abandoning in some cases certain degrees of their *local independence* to the profit of the *situation as a whole* now become the *all important factor*.

This co-ordinated action undoubtedly facilitates the working, harmonises the arrangements to be made, and gives particularly striking results as regards :

— regular working,
— use made of the engines,
— the selection of « when required » train workings,
— the splitting up of trains.

3. *The functions of the Offices of the Heads of Station Movements.*

These Offices have been put into operation in the following stations :

Paris, Villeneuve-Triage, Dijon-Perrigny, Lyons-Perrache, Lyons-Guillotière, Saint-Etienne-Châteaucreux, Nîmes (P. V.).

with a view to :

— regulating the movement of trains,
— speeding up the operations,
— increasing to the maximum the output of these stations having a heavy traffic.

In this case, the local Dispatching System is covered in the important stations as regards the

— *formation* of passenger trains,
— *relief* of engines,
— *shunting* goods trains.

The Head of the Train Movement Office regulates the movements :

— on lines in the station

by co-ordinating on these lines :

— the movement of the incoming trains,
— the movement of the departing trains,
— the shunting of stock,
— the movement of single engines.

These employees also cover liaison duties between :

— the station and
— the depot (or the depots)

by transmitting to the Locomotive Running Department the orders of the Operating Department for the locomotives necessary for « as required » and special trains. They have to take care not to order engines except when the train is complete and ready to start at the time fixed; they will also see that the various sets of men are properly arranged.

On the other hand, they assist the task of the Locomotive Department by delaying the departure of supplementary trains in cases when this will make it possible to make use of foreign engines returning to their home depots.

IIb. — **At the stations and depots.**

The collaboration of the stations and depots has as its object the :

— regularity of the movement of the trains,
— the best utilisation of the locomotives.

In principle, the depots should supply all engines asked for by the stations, but it is the duty of the stations to assist the Locomotive Department so as to utilise, as often as possible, every engine returning to its home depot.

[625 .143.4)]

The chevron fish plate,

By Georges COULLIÉ,
CHIEF ENGINEER, MIDI RAILWAY COMPANY,
and
Louis CADIS,
ASSISTENT ENGINEER, PERMANENT WAY DEPARTMENT OF THE MIDI RAILWAY COMPANY.

Figs. 1 to 9, pp. 59 to 63.

(*Revue Générale des Chemins de Fer.*)

In the May 1927 number of the *Revue Générale* (1) we described the principal features of a new type of fish plate, the chevron fish plate, which appeared to present valuable features from the mechanical point of view on the one hand as regards the joints remaining in good order, and on the other from the electrical point of view by giving good continuity throughout the track. As the Midi Company as an experiment has fitted these chevron fish plates on 1 000 metres (1 094 yards) of track on the Bordeaux to Cette line (between Saint-Médard and Beautiran stations), we have been able to record certain data which have very definitely confirmed the conclusions drawn from the theoretical consideration of the subject This present note describes the results obtained.

I. — Recording the improvement in running conditions.

The Marey apparatus already described in detail in the *Revue Générale* (2) was made use of. We will limit ourselves to a description of its main features, whilst giving more detailed information as to the special conditions under which it was used.

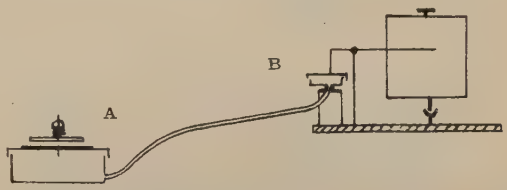


Fig. 1.

The essential part of the Marey apparatus consists of two manometric vessels connected together by india rubber tubes of 5 mm. (3/16 inch) diameter. The box A (fig. 1) is formed by a metal cylinder closed at the top by a rubber membrane. This membrane carries at its centre a metal disc, which in turn carries on its exact centre a small screwed rod : thin lead discs can be placed over this rod, a nut and spiral spring being used to hold them lightly against the disc. The box B, similar to box A but smaller, carries a lever, the movements of which can be recorded on a revolving cylinder.

When placed on a horizontal surface liable to undergo in the vertical plane sudden displacements, the box A at each blow or shock undergoes variations in pressure owing to the elasticity of the

(1) See also *Bulletin of the Railway Congress*, October 1927 number. page 829.

(2) October 1927 number : « Recherches expérimentales des conditions de stabilité des voies en acier. » (Experimental investigations into the stability of steel tracks). by Mr. Couard.

membrane and the inertia of the weight on it. The above arrangement makes it possible to record directly the resulting variations in pressure.

During a preliminary series of observations, four sets of A and B boxes were used. Two A boxes were placed on the joint sleepers of a joint fitted with chevron fish plates : the other two on joint sleepers of a neighbouring joint with Midi pattern fish plates. The four corresponding B boxes were arranged side by

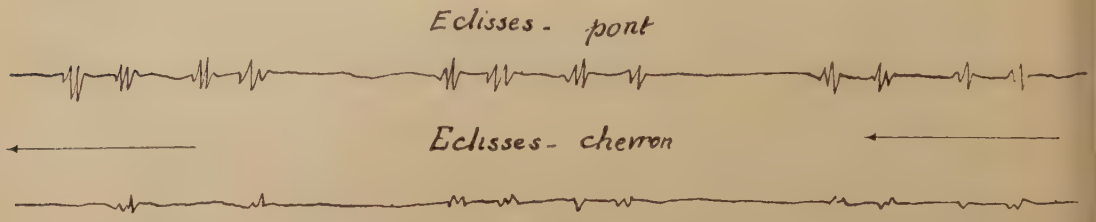


Fig. 2.

Explanation of French terms : Eclisse-pont = Bridge type fish plate. — Eclisses chevron = Chevron type fish plate.

side against the recording drum so that the records were drawn down in parallel. The records given during the passage of a train by one set of A and B boxes is reproduced in figure 2. The two joints under observation were 22 m. (72 feet) apart, the joints with the bridge type fish plates being the first to be run over. The two sets of records are out of line one with the other. On the figure an arrow indicates the direction of travel, thereby making it possible to compare the effects produced at the two types of joint by the passage over them of the same wheel. It will be noticed that the shocks recorded are grouped together in sets of four; the first two shocks are produced in a group by the passage of the last bogie of a vehicle, the last two by the passage of the first bogie of the following vehicle. Whereas the displacements occurring whilst passing over joints with bridge type fish plates are very marked, the displacements at the chevron joints are hardly legible. In order to make sure that the differences were indeed due to the fish plates and not to other causes, the chevron fish plates and the bridge type fish plates were changed over, the packing of the sleepers being left untouched : the further records obtained in every way confirmed those of the first series.

Fig. 3.

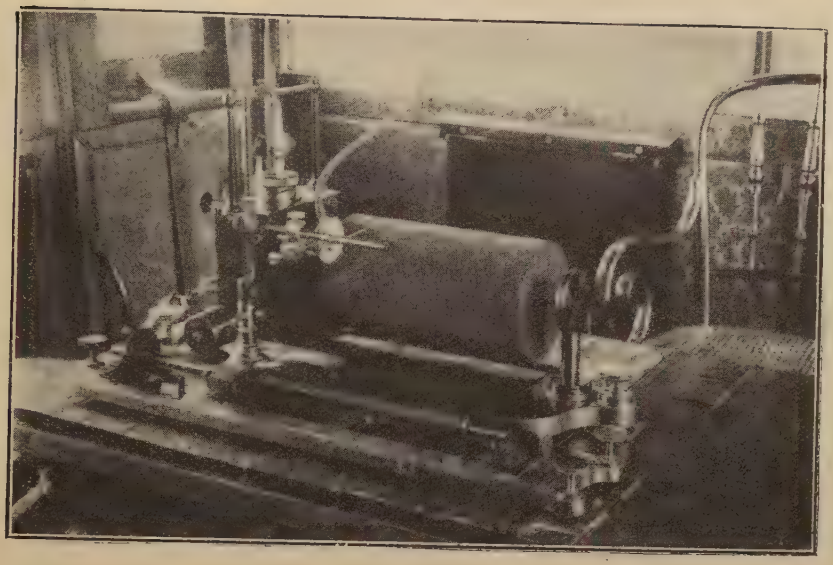


Fig. 4.

Fig. 5.

In a second group of observations, a box A was fastened directly to the axle of a carriage, the box B and the recording drum being carried inside the carriage (figs. 3 and 4). The displacements of the lever were recorded throughout the whole of a 5-km. (3 miles) run at an average speed of 80 km. (50 miles) an hour, the run including the 1-km. (1 017 yards) test section mentioned above. Figure 5 is a reproduction of the records obtained when running over joints with the Midi pattern bridge fish plates, and also when running over the chevron fish-plated joints. On the section fitted with chevron fish plates, the position of the joints cannot be distinguished : on the contrary, the vibrations recorded are rather more marked than on the trial section between two joints. This last result would appear to be attributable to the track being in an indifferent state of maintenance : it is practically independent of the type of fish plate.

II. — **Measurement of resistance.**

The electric conductibility of the joints tested was carefully measured by means of the Ferrié-Lebaupin instrument.

The Ferrié-Lebaupin instrument includes essentially 2 independent millivoltmeters placed behind the same frame, in such way that their needles, whilst coinciding when there is no current passing through the windings,

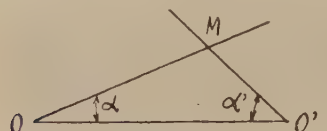


Fig. 6.

turn in opposite directions and cross when a measurement is taken (fig. 6). Every point M of the scale corresponds in this way to a well determined value of the ratio of the intensities of current flowing in the millivoltmeters : in order to ascertain this value quickly, all that is necessary is to have taken care to trace beforehand on the scale some curves of the $\frac{\alpha}{\alpha'}$ = constant family, recording for each the value of the constant. When using the instrument to measure the conductibility of the fish plated joints, one of the millivoltmeters is connected to A and B on opposite sides of the joint to be measured and the other between two points C and D on the rail, and a suitable current caused to flow

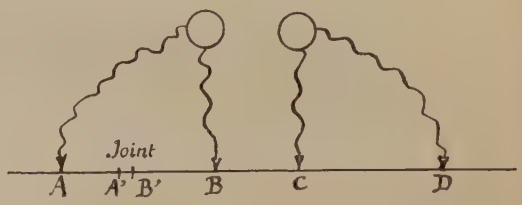


Fig. 7.

(fig. 7). In practice A B is made equal to C D and both equal to 1 metre (3 ft. 3 3/8 in); the apparatus having been set to give by direct reading the value of the ratio :

$$\frac{\text{resistance AA}' + \text{resistance of the joint itself} + \text{resistance BB}'}{\text{resistance CD}},$$

we get directly the resistance of the joint itself expressed in terms of metres of rail increased by one.

On the test section with the chevron fish plates, measurements of the electrical resistance of the joints were taken at different times with the Ferrié-Lebaupin instrument. The results were as follows :

1. In March 1927 after a month in service :

76 joints shewed a resistance equal to or less than 1 metre;
3 joints shewed a resistance between 1 and 2 metres;
1 joints shewed a resistance of 3.5 metres;

2. In June 1927 after four month's service :

33 joints shewed a resistance equal to or less than 1 metre;
24 joints shewed a resistance between 1 and 2 metres;
11 joints shewed a resistance between 2 and 3 metres;
3 joints shewed a resistance between 3 and 4 metres;
1 joint shewed a resistance equal to 5 metres;
8 joints shewed a resistance higher than 10 metres.

3. In September 1927 after seven month's service :

52 joints shewed a resistance equal to or less than 1 metre;

10 joints shewed a resistance between 1 and 2 metres;
1 joint shewed a resistance equal to 3 metres;
3 joints shewed a resistance between 3 and 4 metres;
1 joint shewed a resistance equal to 5 metres;
3 joints shewed a resistance between 5 and 6 metres;
10 joints shewed a resistance above 10 metres.

4. In december 1927 after eleven month's service :

66 joints shewed a resistance equal to or less than 1 metre;
9 joints shewed a resistance between 1 and 2 metres;
2 joints shewed a resistance between 2 and 3 metres;
1 joint shewed a resistance equal to 5 metres;
1 joint shewed a resistance equal to 6 metres;
1 joint shewed a resistance above 10 metres.

There was no correspondance between the joints admittedly unsatisfactory; the number varied from one series of measurements to another, and it was quite exceptional for a joint to be found defective twice. The excessive resistances appeared to be due to some bolts having slacked back a little : in every case it was only necessary to pull them up with a spanner to make the conductibility of the joint almost perfect.

Fundamentally, these imperfections should be taken as being, instead of a cause for uneasiness, a further confirmation of the characteristic features of the chevron fish plate. Obviously it must be admitted that periodic inspection is necessary, but this work cannot be considered as being additional, as for many years it has been found desirable to do it, no matter what the type of fish plate used.

We did not stop at this point however; as the nuts used did not give sufficient guarantee against slacking back, experiments were made with various designs and resulted in the adoption for future use of a spring check nut, particulars of which are given below, which, so far as can be judged from the tests made, will give quite satisfactory results.

The check nut, in steel, 6 mm. (15/64 inch) thick only has a radial slot, 1.5 mm. (1/16 inch) wide cut in it : the inside diameter of the thread is very slightly less than the diameter of the bolt : the

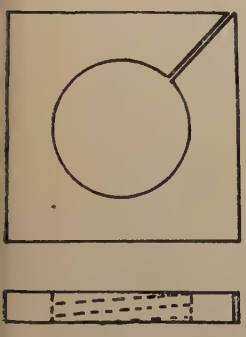


Fig. 8.

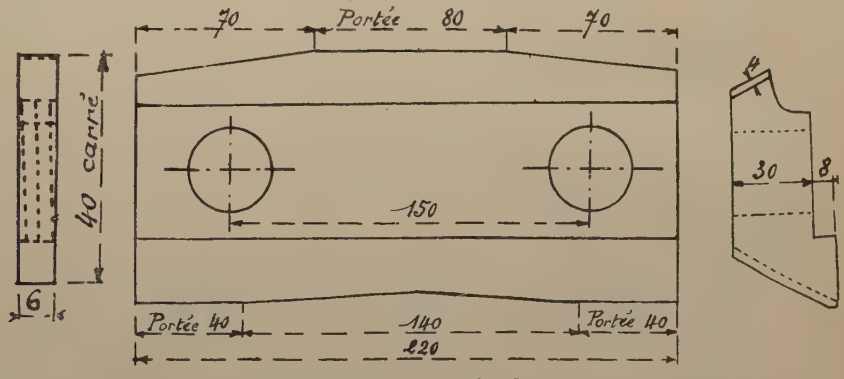


Fig. 9.

Explanation of French terms in figures 8 and 9

Carré = square. — Portée = bearing.

check nut therefore exerts sufficient pressure on the thread of the bolt, taking its inertia into account, to be held immovably in position in spite of any vibrations set up by the passage of rolling loads. Any slacking back of the nut held between the fish plate and the check nut is consequently avoided. A much worn joint in the outgoing line used by locomotives from No. 1 Shed at Bordeaux-Saint-Jean was fitted six months ago with chevron fish plates with bolts having spring check nuts : the electrical resistance of this joint taken on several occasions was always less than that of 2 m. (6 ft. 6 3/4 in.) of rail.

In view of these results, the Midi Company has decided to extend the use of these chevron fish plates to all lines on the system where rails have to be renewed. This decision applies to about 100 km. (62 miles) of single track. Figure 9 shews the final pattern of chevron fish plate adopted.

We have every confidence that when this programme is completed, it will afford a thorough confirmation of the advantages demonstrated by the first tests, the value of which cannot escape the notice of those engineers who are devoting their energies to the improvement of rails-joints. As a simple indication, the economy that can be realised by using chevron fish plates on electrified lines may be valued at 45 francs per single joint, that is to say, for a line laid with 22 m. (72 ft. 2 in.) rails, at 4 francs per linear metre : 45 francs may be taken as the cost of material and labour of the copper bonds which are essential when ordinary fish plates are used but become unnecessary when chevron fish plates are employed : it should be noted that the saving due to the difference in cost of the fish plates has not been taken into account, although the cost is largely in favour of the chevron fish plate.

[625 .113 (.73)]

String lining of curves made easy,

By Charles H. BARTLETT.

Fig. 1, p. 70.

(From *Railway Engineering and Maintenance.*)

It is a well known fact that all railroad curves are made up of circular curves and easements or spirals. The circular curves are staked out by engineers before the track is built, while the easements are more usually installed after the curve is laid. The above statement is especially true of curves which were laid out many years ago, when the use of an easement curve was considered an unnecessary refinement of engineering. It is an equally well known fact that no curve, no matter how well it may be ballasted or how carefully it may be maintained, will remain as it was originally staked out. This change, due to many different forces (such as temperature changes, continual pounding by passing trains, shifting of the roadbed, etc.), produces what are known as « sharp » and « flat » spots in the original curve. By a sharp spot is meant a portion of the curve whose curvature is greater or « sharper » than that of neighboring portions of the curve, or, less commonly, than the curvature as first staked out. Similarly, a flat spot is one whose curvature is less than that of the adjacent portions of the same curve. It is obvious, that such a condition is not desirable, and equally obvious that the sooner it is corrected, the less discomfort will be caused to passengers and the safer the track will be for all trains, passenger and freight. It is the purpose of string lining to correct the defects of alinement and to give to the curve that uniformity which insures both good riding qualities and safety, restoring the curve to its original shape or nearly so.

Although string lining, in one form or another, has been in use on some of the railroads of the U. S. A. for many years, its use has not been wide-spread and many roads have preferred to leave the work of realining curves to the engineering department, or else entirely to the track foremen, each of whom has had his individual way of accomplishing the work.

This practice has resulted in a condition found on almost every road in the country, whereby on one section all curves will be well lined and properly elevated, while on the next section they will be just the opposite — the reason, of course, being the difference, in the ability of the two section foremen to line their curves. Such a condition could be remedied quite easily if a standard procedure for curve lining could be adopted. Naturally it follows that any system adopted as standard must be easily learned and easily remembered by all foremen and supervisors; and, moreover, that such a system must reduce to a minimum not only the time required for calculations, but also the effect of judgment or experience. In other words, it must be nearly « mechanical » in its application. It is claimed for the system described in this article that it possesses

to a high degree all these qualifications. The proof will, of course, rest in the application of the principles of actual conditions.

It should be pointed out at this place that this system is not that commonly known as « cording a curve ». The latter method consists of measuring the middle ordinates to a curve from a cord stretched between points on the curve, averaging what appears to be the predominating ordinate, and then throwing the track, by means of lining bars or similar devices, back and forth until it is approximately uniform. While this method will undoubtedly work, after a fashion, any foreman who has ever tried it knows that he will have to go over the curve several times before a satisfactory result can be obtained, and the labor required and the time involved must surely have struck him as so much wasted effort. Such a man will readily concede that if the proper throw could be made the first time much needless work would be saved. The proposed method points out exactly what correction is needed at each point in order to make a true curve.

Another point which should be emphasized here is that no elaborate equipment is necessary. No piano wire and no delicate measuring instruments are required. All that is needed is a good stout piece of cord, similar to that used by bricklayers or masons, and an ordinary ruler whose divisions start at the end of the rule in order to allow it to be placed against the gage line of the high rail. Moreover, there are no complicated mathematical expressions to use, as the process consists in the addition and subtraction of small numbers representing the middle ordinates as measured. Frequent checks on the work can be made, so that if a mistake is made it will be detected quickly, before the work has gone too far.

Finally, the system *works*. This has been demonstrated in actual practice by re-alining nearly a thousand curves by its use. And it works, not only from the standpoint of the maintenance man, by giving him better track in less time and with less expenditure of labor and money, but also from the standpoint of the engineer, inasmuch as by its use can be determined all the properties of a curve which can be ascertained by using an instrument. That is, the central angle between tangents, the central angles of all spirals, the points of spiral, points of curve, degree of curvature, rate of change of curvature of spiral, etc. — all can be computed readily and easily within satisfactory limits of error. When to this fact are added the many other advantages — such as the fact that the system is from five to ten times as rapid as transit lining, the engineer as well as the foreman will realize that it will pay him to investigate the possibilities of this method of string lining.

The method is rapid

The time required to master the fundamental principles of the entire method varies from two or three hours to about eight or nine hours. One day's working time has always been found sufficient to enable a new man to obtain a thorough understanding of all the rules and necessary operations.

The various operations in setting stakes for a curve, such as taking the original data, figuring the curve, and setting the stakes, have all been carefully timed. It takes, on an average, about 1/4 minute per station to take the original ordinates of a curve. This in an average figure, and takes into account delays due to waiting for trains to pass, walking between stations, rechecking any doubtful ordinates, and finding the point of ending and beginning of the curve.

It takes, on an average, about 1 1/2 minutes to distribute stakes at each string lining station (from the original bundles), drive an iron pin into rock

ballast, shake the pin to make a good hole, drive a wooden stake, and set the tack in the stake. In cinder ballast the average time per stake set in the field is about 1 to 1 1/4 minutes, depending upon the firmness and density of the ballast. This makes the total time required to perform all the field operations in lining a curve equal to about 1 3/4 minutes per stake in rock ballast, and 1 1/4 to 1 1/2 minutes per stake in cinder ballast.

Several trials are required.

It is obvious that in attempting to line a curve which is badly out of shape, several trials will be required before the best curve can be found. If a foreman and his gang of men set out to reline a curve by eye only, several attempts will be required before a satisfactory curve can be obtained; and the same is true of the computations required by the proposed system, the only difference being that instead of the actual track on the curve being changed, only the figures representing the curve are changed. It has been found, consequently, that an ordinary school slate, ruled into columns, furnishes the easiest and most satisfactory way of making the changes required, inasmuch as it requires more time to make erasures from a sheet of paper. The length of time required to make the computations on the slate will, of course, depend upon the individual and upon the number of curves he has lined. Curves requiring no more than a foot throw either in or out, can ordinarily be lined upon a slate at the rate of from 1 to 2 stations a minute. The author has lined (on the slate) curves of 86 to 90 rails in 20 and 30 minutes time, whereas some harder and shorter curves which were badly out of line have taken as much as three or four hours, in order to obtain a curve which would not throw beyond a certain fixed limit. The figure of 1 to 2 stations a minute includes the entire time required to line the curve on the slate, from the time of starting until the curve is fully lined; it includes making all changes, erasures, etc.

Thus, according to the above average figures, a curve in rock ballast, a mile long, would require about seven hours to re-line, from the time of starting in to get the original data until the last stake was set. As a matter of record, several curves a mile long have been lined by this method at the rate of six hours per mile. All of these curves were compound curves (before lining), and were spiraled. In relining them by the string system, proper spirals were placed at each end and between the branches of the compounds. The author's experience in lining curves with a transit has led him to believe that the time it would require to line a compound curve (with two branches), spiral both ends and insert a spiral between the branches of the compound, and drive a stake every 33 feet in rock ballast, would be anywhere from three to five working days of eight hours each. On such a curve (a mile long), the intersection angle could not be run, in all probability, and the matter of lining an entire mile without « running off the embankment » would be quite a problem.

In addition to the fact that a curve can be figured and the stakes set much more quickly by the string lining method than by the transit method, there are other advantages which make for a considerable saving in time. For example, passing trains and motor cars do not cause any serious loss of time. With an instrument, every time a train or motor car goes by, a new set-up must be made, which takes from three to eight or ten minutes, depending upon how far the flagman has to go for a second backsight. With the string lining method, the only time lost is that which it takes the train or motor car to pass the working point. This is an advantage of considerable weight on heavy traffic lines.

String lining is cheaper than transit lining because of the time saved, because fewer mistakes are made, because a better curve is obtained, and because the throws are kept to a minimum. In transit lining, a party must necessarily consist of an instrumentman, a rodman and a chainman. The combined salaries of these men will equal about two to three times that of a chainman and two section laborers for the same length of time. Adding to this fact the saving in time effected, we have, considering string lining as say four times as rapid as transit lining, the cost in engineering labor to use the string method as approximately one-eighth of that with the transit method. These figures, if anything, are conservative. The actual saving will run as high as 9/10 in most cases. Not only are the salaries of the instrumentman and rodman saved on each curve, but these two men are then available for other work in the engineering department.

Facilitates compounding.

Further, string lining permits compounding a revised curve slightly for a given distance, usually quite short, in such a manner that the saving in throw effected is marked. This is a valuable property for the maintenance man, since it enables him to install a smooth-riding curve with a minimum of throw, and consequently, with a minimum of time and labor. The time of a section gang can be cut down by several hours in this way, thus realizing a big saving. Suppose that at string lining station number 10 of a curve, we change a revised ordinate by 1 (tenth of an inch) in order to decrease the throws, then, at the end at some such station as 67, say, we have effected a saving in throw of twice 67-10 or 114 tenths of an inch (11.4 inches); yet we have compounded the curve only 5 1/4 minutes and for a length of only one rail. It would never have occurred to an instrumentman with a transit to make such a compound, and he would have carried his curve through to the end, with the consequent big throws. String lining, then, permits of keeping the track close on the old bed (a big advantage) and yet obtaining a smooth-riding curve.

By making it easier for the trackman to line his curve (by placing the stakes every 33 feet instead of every 50 or 100 feet), we avoid the small sharp and flat spots which creep in because of the fact that a foreman has to line between the 100 feet transit stakes by his eye. We accordingly prevent the curve from getting so quickly into bad line once more, and thus effect another saving in maintenance cost.

Work can be inspected in advance.

Since a record of the throws required at each joint or string lining station to line a curve into a new and better shape is obtained from the slate or calculation paper, the operator has a record, in black and white, of how much he proposes to throw the curve. He can take this record to the roadmaster or assistant engineer, as the case may be, to look over at his leisure and inspect the changes necessary.

If the work were to be done with a transit, the instrumentman would first have to set spikes or stakes, and then go to every transit station and measure the throw, if such a record were to be given his superior officers before the curve was actually lined in the field. The advantage of having such a record as string lining gives is readily understood by anyone who handles such matters on a railroad. The size of the gang required to do the work of lining, the approximate amount of time required to do the work, and all other arrangements or information relative to the task of re-alining the curve are known and determined before a start is made.

String lining permits of so compound-

ing a curve (if necessary, of course) that the throws at or near a permanent track structure can be made either zero or negligible. This also is a means of avoiding a great deal of unnecessary work and of saving time and expense. Such small throws as are required can be made upon the slate, so that the entire amount of shifting of such obstacles can be seen by the man in charge of moving them before the curve is lined. To do this with a transit would require a great deal of time and the running in of many trial curves before a final curve is selected.

The equipment necessary to do string lining is much simpler and much cheaper than that required with a transit. In addition to this, it is easier to carry around, no care need be felt for its safety or adjustment; and, finally, expensive errors in instrument work done by more or less inexperienced men are entirely avoided.

Summary.

The principal advantages of string lining, then, are as follows :

1. It is much cheaper than any other method.
2. It is much quicker than any other method.
3. It is much easier than any other method.
4. It is easily learned, easily remembered, and easy to use.
5. Errors are readily detected by several checks on the work.
6. Throws can be governed almost at will.
7. Spirals are no longer an « affliction » to install, but are so easy and help to decrease the throws required to line a curve by so great an amount that their installation becomes easy and automatic.
8. A record of proposed changes or throws is obtained and is ready for inspection before any work is done.
9. It lowers maintenance costs.
10. It permits of relining curves annually at small expense of time and labor.
11. It gives a more satisfactory curve.
12. No expensive or easily breakable equipment is required.
13. Less men are needed to do the work of figuring the curve.

. . . . . .

Now, let us assume that it has been decided to change an actual curve in certain ways so as to rectify any errors of curvature that may exist, such as sharp and flat spots. In the following discussion, no mathematical analysis is given for any of the principles stated, except the simple geometrical proof for the theorem regarding the throwing of a joint or station, although complete and satisfactory proof can be given mathematically for all of the rules given.

Sum of ordinates must remain constant.

The first principle of string-lining is that the sum of the ordinates of a curve must remain constant throughout any series of operations designed to correct the alinement of that curve. In other words, if a curve is measured and the ordinates taken in the field total 656 tenths (or eighths, quarters or any other units) of an inch, the total 656 must remain the same for any revised curve. It can be proved that this total represents the actual angle between the two tangents or pieces of straight track at each end of the curve; and inasmuch as it is obvious that the angle between the straight tracks cannot be changed, the total of the measured ordinates must not be changed.

The second principle is that the sum of the errors between the figures for the original curve and the figures for the revised curve must equal zero, the error at any station being defined as the difference between the original ordinate

and any revised ordinate selected by the man lining the curve. It is, briefly, the original ordinate less the revised. Suppose that at some station of a curve the ordinate measured in the field is 48 eighths of an inch, or 6 inches. If it is desired to change this ordinate when re-lining the curve, to some such figure as 41 eighths of an inch (or 5 1/8 inches) the *error* at that station will be 48 less 41 or 7 eighths of an inch. If the revised ordinate is larger than the original, the difference or error is termed negative and a minus sign placed in front of it. For example, if the revised ordinate in the above case happened to be 53 instead of 41, the error would be 48 less 53 or minus 5 eighths of an inch. In view of the above explanation, it will be clear to the reader that, if the total of the revised ordinates must equal the total of the original ordinates, the total difference between the two must be zero, the total difference being merely the sum of the separate differences or errors at each station. In other words, the second principle is practically the same as the first, but is merely a more convenient way of expressing it.

Effect of throw on adjacent stations.

The third principle is very important, and should be thoroughly understood before the reader attempts to line any curve. It is as follows :

Rule : If one joint (or station) of a curve is moved in or out a certain distance (which distance is called the throw), the middle ordinates at the stations on each side of the one moved will be changed by half the amount of the throw, and in the direction opposite to the throw.

The above rule is best explained by a diagram. Let the reader refer to figure 1.

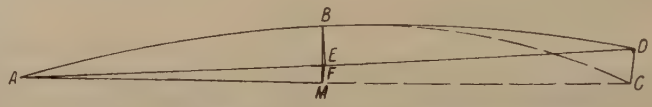


Fig. 1. — The effect of throw on adjacent stations.

in which is shown a portion of a circular curve ABC, whose chord is AC and middle ordinate MB. The arc ABC can be taken, if desired, as the length of two rails of the high side of a curve, with AB as one rail and BC as the other, with the joint at B. It is plain, therefore, that as a practical matter, the rail BC can be moved without disturbing the rail AB. Therefore, consider that the joint (or station) C is moved or thrown out — that is, away from the center of the curve — to the point D, so that the rail BC now occupies the position BD. The new chord (or line representing the position of the string) is now the line AD, and the new middle ordinate is BE. In actual practice, the length BE is so closely equal to MB (the original middle ordinate) less MF that the difference is negligible. But MF, from geometry, is almost exactly one-half of CD, the amount of throw. This is because in the triangle ACD, M is the middle point of AC, and therefore AM is one-half of AC, so that MF is one-half of CD. Therefore, BE equals MB less MF; or, in words, the ordinate *after* throwing equals the ordinate *before* throwing, less one-half of the throw. In the same way, it can be shown that the joint (or station) the other side of C (on the right) is similarly affected. Hence, an out throw at any joint increases the ordinate at the joint by the full amount (CD in the diagram) and decreases the ordinates at each of the joints on the two adjoining sides by one-half the amount of the throw. Con-

versely, if the throw is in, the ordinate at the station thrown is decreased by the full amount of the throw, and the ordinates at the stations on each side are increased by one-half the amount of the throw.

From the above demonstration it is seen that the operation so described consists in the *addition* or *subtraction* from the measured middle ordinate at a station of certain quantity, known as the throw, and in the *subtraction* or *addition* of half that amount to the ordinates at the two stations on either side of the one thrown.

A simple example.

For example, suppose that at any three consecutive stations or joints of a curve we have the following three ordinates :

18 24 18

These indicate a typical sharp spot, which of course, it is desired to eliminate by making all three as nearly equal as possible, and thus making a uniform curvature. We can « throw » the middle joint in by subtracting a certain amount and adding half as much to the two figures 18 on each side. Note : It is obvious that the 24 must be reduced and the two 18's increased, so that all three can be made equal.

We can subtract 4 from 24, which will add 2 to both figures 18. We then have:

18	24	18
+2	—4	+2
—	—	—
20	20	20

Thus, we have equalized all three ordinates and made this section of the curve a true circle, which is one of the purposes of string-lining.

The process of lining a curve (on a slate or a sheet of paper) consists in repeating the above operation at every station, with certain obvious modifications. It is logical to conclude that from the simplicity of the above operation there must be some definite relation between error and correction; and, in fact, such is the case. However, it is not our purpose to give here a detailed mathematical analysis, but simply a set of working rules for the performance of the mechanical operations of string-lining.

In connection with the above rule, it should be noted that the throws at the first and last stations of a curve must be zero. If the first or last stations are thrown, the resulting half-throws will be imparted to the tangent or straight track, throwing it out of alinement. For example, if the first station of a curve has an ordinate of 0, and we throw the track 2 (units of any kind) out, the straight track at the left of the 0 will have a half-throw of -1, which will put a small kink in it. Consequently, we must correct the first station by throwing the second, making the throw such that the half-throw at the first station will correct it.

Relation between error and throw.

If, at each station, we add all the errors to and including that station and write them down opposite their station numbers, we obtain a column which we can call the *Sum of Errors*. If now we obtain the sum of all of these from the first station to any given station, and write that sum opposite the *following* station, we obtain the half-throw required at that following station.

Let us suppose a curve having 53 stations, opposite each of which is written an actual and a revised ordinate, in two columns, and in the third column the differences, or errors, between the actual and the revised ordinate at each station. Let us now put down opposite the first station the error at that station; opposite the second station, the sum of the

first and the second errors; opposite the third station, the sum of the first three errors; and so on, until at station 53, we have the sum of the errors for the entire curve, which must, of course, equal zero. The addition of the errors is algebraic; that is, due regard is taken of the sign of the error. For example, if the first error is (5), the second (-3), and third (-6), the sums of the errors at the first three stations are, in order (5); (5) + (-3) = (2); and (5) + (-3) + (-6) = (-4). It is, of course, not necessary to add all the errors every time, since the sum at any station is obtained directly by adding to the sum of the errors at the previous station, the error at the station. For example, the above would be obtained actually as 5; 5-3 = (2); 2-6 = (-4), etc., etc.

Having obtained the column headed « Sum of errors », we next add this column to and including each station and bring the total into the next column under the *following* station. To illustrate (remembering that the throw at the first station is zero), we have for the above figures, the following half throws :

First station	0
Second station	0+5= +5
Third station	0+5+2= +7
Fourth station	0+5+2—4= +3

Once again, it is not necessary to add the figures from the first station on, since the same result is obtained more easily by adding to the sum previously obtained the adjacent figure in the next row. Example :

First station	0
Second station	0+5= +5
Third station	5+2= +7
Fourth station	7—4= +3

A résumé of the above operations may make the process clearer. Consider the following hypothetical curve :

Station or joint number.	Actual original ordinate in tenths of an inch.	Revised ordinate for re-alined curve.	Error.	Sums errors.	Half throw.
1	1	0	+ 1	+ 1	0
2	3	2	+ 1	+ 2	+ 1
3	7	4	+ 3	+ 5	+ 3
4	3	6	- 3	+ 2	+ 8
5	4	8	- 4	- 2	+10
6	7	8	- 1	- 3	+ 8
7	10	8	+ 2	- 1	+ 5
					+ 4

In the first column are the station numbers.

In the second column are the middle ordinates of the actual curve, measured in the field.

In the third column are the revised ordinates selected for the curve.

In the fourth column are the errors, *e*, obtained as explained above. For instance, at station 3, 7—4=3; at station 5, 4—8=—4; etc.

In the fifth column are the sums of the errors to and including each station. These are obtained by writing down the error at the first station, and then at each station thereafter writing down the sum of the preceding figure and the figure on the next line in the column headed « Error ». Thus writing down 1 in the first station, we next add 1 and the 1 shown on line 2 as error at station 2. This makes 2. To this 2, we add the 3 on the next line below, in the error column, making a total of 5.

In the sixth column are the half-throws. To obtain these we add horizontally instead of diagonally downward as we previously did to obtain the sum of the errors. Thus, 0 + 1 = 1, at station 2; 1 plus the 2 at station 2, is carried as 3 and written under station 3; 3 plus the 5, at station 3, is written as 8 at station 4, etc. The arrows indicate the direction of the additions and the position of the results.

In this way a final half-throw is obtained at the last station. This half-throw must be zero, as we have seen above. However, because it is nearly impossible to pick the correct ordinates at the first trial, this half-throw will not generally be zero, and we must make it so.

Method of making half-throw at last station equal to zero.

It can be shown that the sums of the products of the error at each station by that station number must be equal to zero, which is a condition that the curve is lined. That is, if at a given station the error (difference between the actual and the revised ordinate) is -5, and the number of the station is 21, the product of these two quantities, is -105. The sums of all such products for the entire curve must equal zero as a condition that the curve is really lined.

It can likewise be shown that the sum of all these products is equal to the half-throw at the last station, whether that half-throw is zero or not. This being the case, it is immediately evident that if the final half-throw is not equal to zero, it can be made so by making equal to zero the sums of the products above referred to.

If we have a certain curve, of any number of stations, and we find that the final half-throw is not equal to zero, but some such number as, say, 46, let us consider a way of changing that 46 to zero. Let us say that the ordinate at the 5th station is 23 on the original curve and 25 on the revised curve. The difference is -2; the product of -2 and the station number, 5, is -10. Now let us say that the ordinate at some other station, as the 37th for instance, is 31 on the original curve and 26 on the revised curve. The error is + 5; + 5 times 37 = + 185.

If, now, we change the revised ordinate at station 5 from 25 to 24, and that at station 37 from 26 to 27, we have kept the total of the ordinates the same, for we have subtracted one at station 5 and added one at station 37. But what effect has this had on the products of the error and station number? The error at station 5 is decreased from -2 to -1, making the product now -5 instead of the previous -10; at station 37, the error is decreased from + 5 to + 4, making the product now 148, instead of the previous 185. Thus, while maintaining the total of the ordinates of the revised curve the same, we have changed the total of the products from the first + 175 (-10+185) to the second, 143 (148—5); that is, we have *decreased* the final half-throw by an amount equal to 175—143, or 32. This leaves the final half-throw now 14 (46 -32). The same process can now be repeated, the only difference this time being that the station numbers of the ordinates changed must be different. The reader will note that the net difference in the sums of the products, which amounts to 32, is the difference between the station numbers of the revised ordinates which were changed; that is 32 = 37 -5. This will always be the case. Consequently, to diminish the final half-throw by the remaining 14, we have but to choose two stations whose numbers differ by 14, and repeat the above process, with the assurance that the final half-throw will now be zero, and the curve will be lined.

The rule for changing the final half-throw to zero can now be announced. It is :

When the final half-throw is positive, subtract from the revised ordinates hav-

ing high station numbers and add an equal amount to the ordinates having low station numbers, choosing stations in pairs such that the sum of the differences of the station numbers taken in pairs equals the numerical amount of the final half-throw. When the final half-throw is negative, reverse the procedures, subtracting from the ordinates, having low station numbers, and adding to those having high station numbers.

Thus, if we have a curve of 63 stations, wherein the final half-throw, obtained with a trial set of revised ordinates, is 59, we could add 1 to the revised ordinate at station 6 and subtract 1 from the revised ordinate at station 48, thereby making a diffence of 48-6 or 42, in the final half-throw. We could then repeat the process, choosing this time stations 8 and 25, whose difference is 17, which difference, added to the previously obtained 42, makes the total of 59 and reduces the final half-throw to 0.

The reader will note that the only condition attached to the above method of change is that the sum of the differences of the station numbers of the revised ordinates changed must equal the final half-throw. The operator, therefore, is wholly at liberty to choose the stations which he will change. This is a valuable property of the system — one which cannot be over-emphasized, as its understanding enables the operator to obtain a much better solution than he otherwise would be able to achieve.

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[621 .132.3 (.494) & 621 .134.3 (.494)]

"Winterthur" type of high pressure locomotive with a boiler having a working pressure of 850 lb. per square inch,

By J. BUCHLI,

ENGINEER,

DIRECTOR OF THE SWISS LOCOMOTIVE AND MACHINE WORKS AT WINTERTHUR.

Figs. 1 to 24, pp. 77 to 92.

(*Schweizerische Bauzeitung.*)

« *Note by the Editor of the Schweizerische Bauzeitung* »: In May 1925, we published the opinions of Mr. J. Buchli, Engineer, Director of the Swiss Locomotive Works at Winterthur, and, Mr. K. Wiesinger, Engineer, Professor at the Polytechnic School, on the prospects of successful working of a locomotive using steam at 850 lb. pressure. In his proposal, Professor Wiesinger estimated the saving of coal at 50 % of that used by the most efficient steam locomotive in service. Mr. Buchli queried the possibility of obtaining so great a saving, his calculations making the figure about 25 %. This theoretical disputation, in which two assertions confronted one another, was not followed at the time by any result, in spite of Professor Wiesinger's wishes. We were of the opinion that it was not possible to arrive by this method at any definite conclusion, and, as by December 1924 Professor Wiesinger had promised to give us a detailed report on this invention as soon as the results of the practical tests were available, we thought it better, in accordance with our customary practice, to await the results of these confirmatory tests. The period when they should take place, furthermore, did not seem too far off, as in 1924 the « Rheinische Metallwaren- und Maschinenfabrik » of Dusseldorf had decided to build an experimental locomotive of 2 000 indicated horse power to the designs and patents of Professor Wiesinger. Unfortunately, this firm, as a result of the industrial crisis in Germany, had to close its locomotive section and abandon the idea of building the locomotive in question. At a later date, Professor Wiesinger published particulars of his invention and his proposals in an illustrated article in « Glaser's Annalen » of 1 March 1927, whereby they were brought to the notice of all interested circles.

In the meantime, the Swiss Locomotive and Machine Works at Wintherthur, designed, built and tested a locomotive at 850 lb. pressure. In the following article Mr. Buchli gives particulars of this new type of locomotive with the results of tests.

Naturally we have no intention — and we should much regret any such inference being so drawn from the above notes — of personally taking position in the controversy, or of favoring one or other of the two parties : we have there-

fore suggested to Professor Wiesinger that he also should put before our readers the essential features of his type of locomotive. If in this we depart from our usual rule of never giving a detailed description of new designs until after they have been tried out, the reason is to be found in the special circumstances of the case, and especially in our desire to leave to the reader competent to do so the task of judging for himself from the information he will have before him, if, as Professor Wiesinger thinks, the practical results obtained by the Swiss Locomotive Works are due to the lines of development he indicated. We think we shall so provide in a correct manner, and, above all, in a more satisfactory form for our readers, a conclusion to the dispute suspended in 1925.

* * *

In recent years, the steam locomotive has found itself confronted by two rivals : the electric locomotive and the Diesel locomotive. The electric locomotive has already made for itself an important position in the traction world, whereas the Diesel locomotive has not yet got beyond its period of evolution, although it will undoubtedly in time become a formidable competitor. Furthermore, during the war, new locomotive building works were built more than able to meet all demands for steam locomotives, which resulted in the sale price of locomotives falling in most cases almost below the cost price. For these reasons, the steam locomotive builder, from having for many years controlled the market for locomotives, is now obliged to direct his efforts into new channels.

The Swiss locomotive industry suffered especially through the situation briefly described above, as it is also obliged to import its raw materials from abroad and to pay high rates of wages. In order to increase the export trade, a matter of vital importance, it is therefore necessary that, apart from perfect workmanship, it should draw largely on the scientific knowledge so highly developed in Switzerland, so as to produce new designs and perfections which would enable it to find an outlet for its products at remunerative prices. It is one of these objectives that the Swiss Locomotive and Machine (S. L. M.) Works is following up by the introduction of high pressures on steam locomotives.

In view of the admitted fact that increased boiler pressure is especially advantageous in the case of non-condensing engines, the S. L. M. directed its whole attention to this aspect of the question. A high pressure locomotive (fig. 1) it has just built has given results which lead to hopes of a new evolution of the locomotive, as the « Winterthur » system has not only confirmed, and in fact exceeded, the estimated savings of coal and water, but the locomotive is extremely simple as regards handling, and its general layout is such that the cost of construction will ultimately, as the result of further improvements, be brought to the level of an ordinary steam locomotive.

The high pressure « Winterthur » locomotive already briefly described in the columns of this journal (1), is the first to use high pressure steam only. The boiler and the engine are designed on new principles, entirely different from the usual practice.

Figure 2 shews by curves *a* and *b* the variation in the thermal efficiency as a function of the steam pressure when exhausting and when condensing. The steam locomotive of the usual type works with boiler pressures of 12 to 18 kgr. per cm² (170 to 255 lb. per square inch) ; if the pressure is raised say from 15 to 50 kgr. per cm² (213 lb. to 710 lb. per

(1) See *Schweizerische Bauzeitung* of the 3 March 1928.

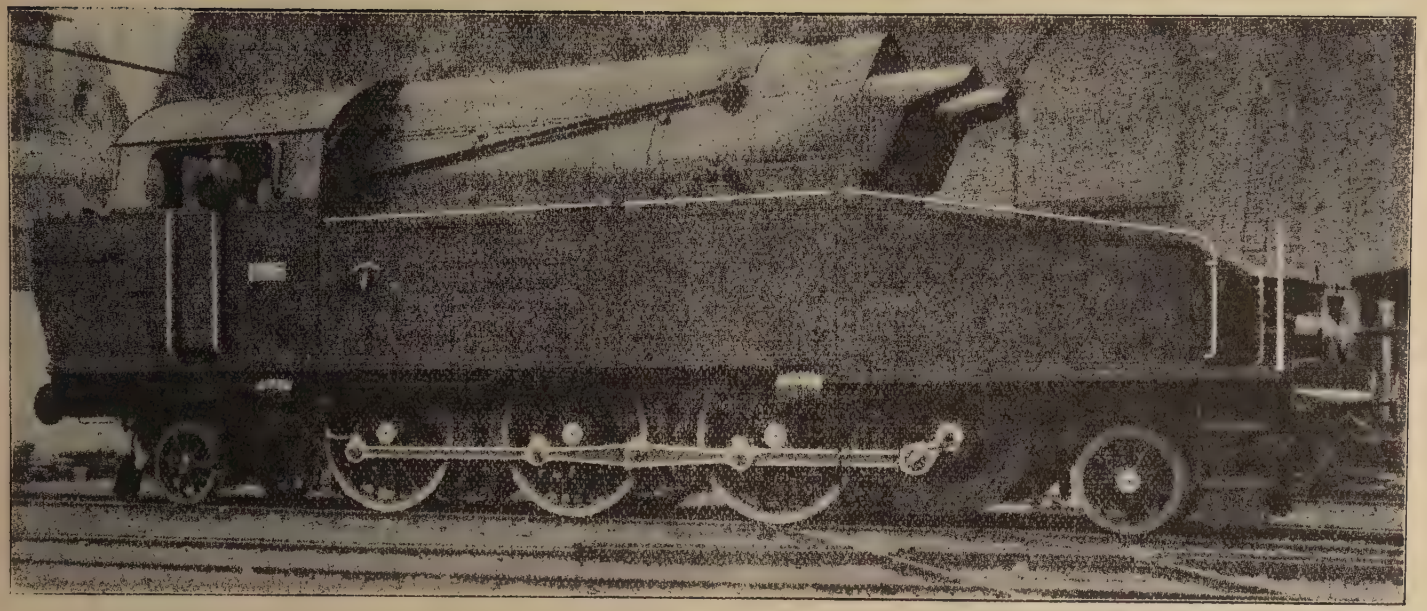


Fig. 1. — « Winterthur » high pressure locomotive with a boiler having a working pressure of 60 kgr. per cm² (850 lb. per square inch), built by the Swiss Locomotive and Machine Works, Winterthur.

square inch), the thermal efficiency increases from 18 to 24 %, whilst an increase from 50 to 100 kgr. per cm² (710 to 1 420 lb. per square inch) gives a further improvement of 27 %. The first thing therefore the builder has to decide is the steam pressure that offers the greatest technical and commercial advantages. Our investigations have demonstrated that this pressure lies between 50 and 70 kgr. per cm² (710 and 1 000 lb. per square inch). The upper curve of figure 2 shews that when condensing and

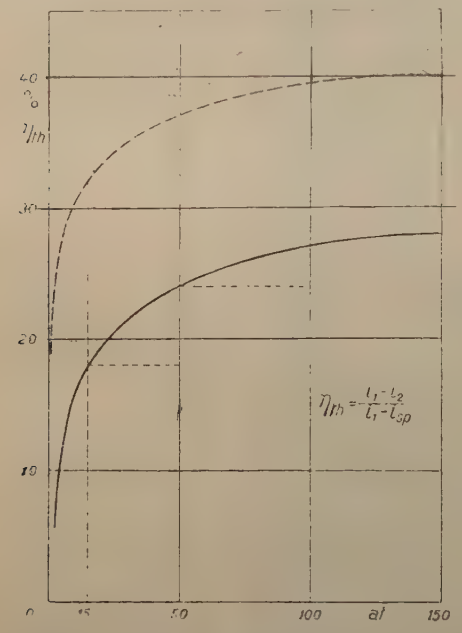


Fig. 2. — Thermal efficiency in relation to the pressure.

Explanation : —— Exhausting engine. — - - - - Condensing. — i_1 i_2 = Heat of the steam before and after expansion. — isp = Heat in the feed water.

with a good vacuum, theoretical savings of considerable importance, as compared with running non-condensing, can be obtained, but the cost of manufacturing the locomotive and the large steam consumption by the auxiliaries form a serious obstacle to the commercial use of this improvement. Undoubtedly the condensing locomotive has the advantage over all other types that the feed water circulates continuously, and the boiler is therefore kept free from scale. But where water plays a preponderating role, the Diesel will ultimately have to be used.

With the increase in the boiler pressure, the difficulties in the way of building the boiler and the engine increase, as do those preventing the steam being highly superheated, for with piston engines the temperature can hardly be carried beyond 400 to 410° C. (750 to 770° F.). When the boiler pressure is 150 kgr. per cm² (2 130 lb. per square inch), the temperature of the water in the boiler reaches the high figure of 370°C. (700° F.), so that for superheating there is only a margin of 30 to 40° C. (55 to 70° F.). For the « Winterthur » high pressure locomotive a pressure of 60 kgr. per cm² (850 lb. per square inch) has been selected.

The layout of the boiler and the engine in relation to the frame and running gear of the locomotive is shewn in figure 3. The boiler is short and high so that there is room in front of it for the steam engine all parts of which have been made accessible and which drives, through gearing and connecting rods, the six coupled wheels of the locomotive. The articulated rods on the jack shaft drive the leading coupling rods at their centres : at the same time it is essential there should be some play where the rods surround the leading crankpins. Thanks to this arrangement the connecting rods are very long, so that the deflection of the springs practically speaking does not affect the smooth running of the engine.

The maximum number of revolutions of the motor is 700 per minute, which corresponds to a piston speed of 8 m. (26.25 feet) a second and to a locomotive speed of 80 km. (50 miles) an hour. It should be noted that at this speed the en-

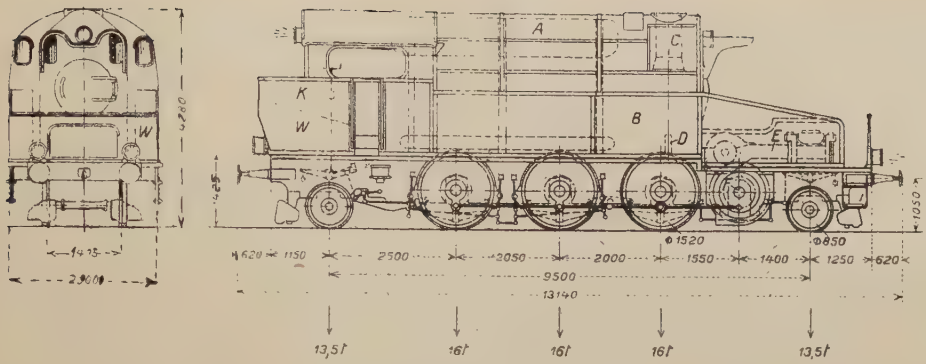


Fig. 5. — Diagram of the « Winterthur » locomotive. — Scale 1 : 150.

EXPLANATION :

A = Upper drum. — B = Smoke box. — C = Chimney. — D = Blast pipe. — E = Steam engine.
F = Jack shaft. — K = Coal. — W = Water.

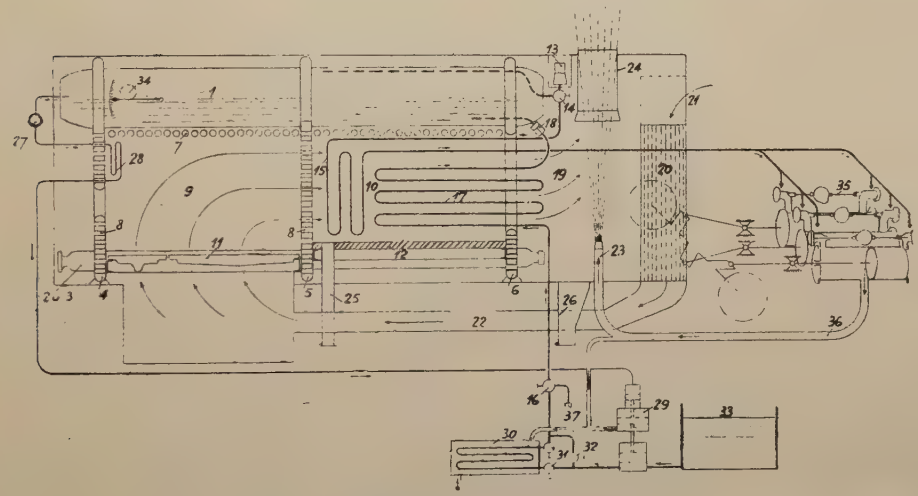


Fig 4. — Diagram of the working cycle.

EXPLANATION :

1 = Upper drum.
2 and 3 = Lower drums.
4 = Back water wall of the firebox.
5 = Front water wall of the firebox.
6 = Front water wall of the boiler.
7 = Tubes.
8 = Tubular stays.
9 = Firebox.
10 = Superheater and feed water heater chamber.
11 = Grate.
12 = Firebrick bottom lining.
13 = Safety valve.
14 = Regulator.
15 = Superheater.
16 = Feed delivery pipe connection.
17 = Combustion gas feed water heater or economiser.
18 = Clack box.
19 = Smokebox.
20 = Air preheater.
21 = Air openings to preheater.
22 = Air channel.
23 = Blast pipe.
24 = Chimney
25 and 26 = Emptying out pipes.
27 = Stop cock for the auxiliary steam supply.
28 = Superheater for feed pump.
29 = Feed pump.
30 = Exhaust steam feed water heater.
31 = Stop cock, normally open.
32 = Stop cock, normally shut.
33 = Water tank.
34 = Water level float.
35 = Steam engine.
36 = Exhaust pipe.
37 = Tank filling hole.

gine is very steady. By suitably choosing the ratio of the gears, the same engine can be used for different working conditions. The engine consequently can be designed independently of the frame and be tested separately as a stationary engine.

Figure 4 illustrates the working of the whole machine. The heaters 30 and 17 may be noted as a main feature. This is the first case in which the feed water before being delivered into the boiler, is heated to a temperature approaching that of the boiler water. The heater 30 is heated by part of the exhaust steam whereby the feed water temperature is raised 80 to 90° C. (175 to 195° F.); in the economiser or hot combustion gas heater, 17, the temperature is further raised to 250° C. (480° F.). After being heated to this temperature, the water is fed into the boiler where it mixes with the water in the boiler, the temperature of which is 270° C. (520° F.). This procedure was expected to separate out all solid elements in the feed water in the heaters, and the service tests have shewn that this is what occurs. During the whole of the tests, both stationary and running, only small quantities of a light soft mud have been deposited in the boiler itself, and can be easily sludged out : after 400 hours in service no traces of scale were found. All parts of the heaters can be cleaned mechanically as will be explained later on, so that this system has many especial advantages in hard water districts.

The new type of boiler would also be suitable for the lowest pressures, as well as for pressures as high as 100 kgr. per cm² (1 420 lb. per square inch). A further point of importance is that the combustion gases before leaving the boiler are caused to scrub the cooler parts of the economiser, and by this means are reduced to as low a temperature as possible, the remaining heat being finally used to preheat the combustion air. The fullest use possible is thereby made of the combustion gases; the system has resulted in the efficiency shewn in figure 5.

The superheater is fitted between the heater 17 and the centre cross water wall, that is to say, at a point where its weight can be most reduced, and where it is most effective. To get an increase in steam temperature from 270 to 400 or 410° C. (520 to 750 or 770° F.) only 20 m² (215 square feet) of surface is needed when the temperature of the gases is 800° C. (1 500° F.). The vacuum in the smokebox is produced directly in the normal way by a blast pipe 23 and the exhaust steam.

The functions of the auxiliaries are clearly shewn in figure 4 and need not be discussed. Two high pressure feed pumps deliver the feed water into the boiler; for pressures exceeding 35 kgr. par cm² (500 lb. per square inch) the usual type of injector cannot be used. The compressed air pumps and the train heating use low pressure steam. A special reducing valve made for this purpose reduces the pressure to 12 kgr. per cm² (170 lb. per square inch).

Boiler. — Before proceeding with the construction of the boiler, the various parts, and especially the different assembled units, were manufactured, and were thoroughly tested in every way during many months, details being altered as necessary until the most rigorous demands were satisfactorily met. The result was that the complete boiler was free from even the slightest leaks when first tested under a hydraulic pressure of 100 kgr. per cm² (1 420 lb. per square inch).

As a first principle, the boiler ought to satisfy the condition that all its parts be so arranged that under the high temperature of the water in the boiler, they can expand freely without any constraining stresses in the material being set up; this is a property that the usual locomotive boiler unfortunately does not possess, so that it is out of the question to use it for very high pressures. The

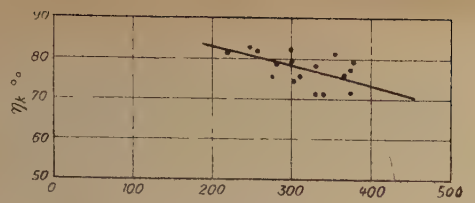


Fig. 5. — Efficiency of the boiler starting from the temperature of the water in the tank.

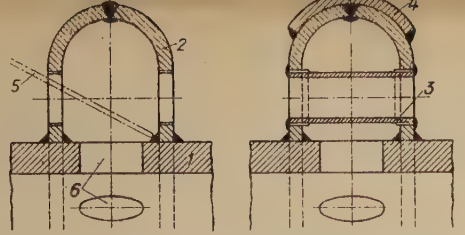


Fig. 7. — Method of assembling the drums and the water walls of the boiler.

1 = Drum.
2 = Cross water wall.
3 = Tubular cross stays.
4 = Höhn joint covering plates.
5 = Welding electrode.
6 = Communicating opening between the drums and the water wall.

The welds are shewn in black.

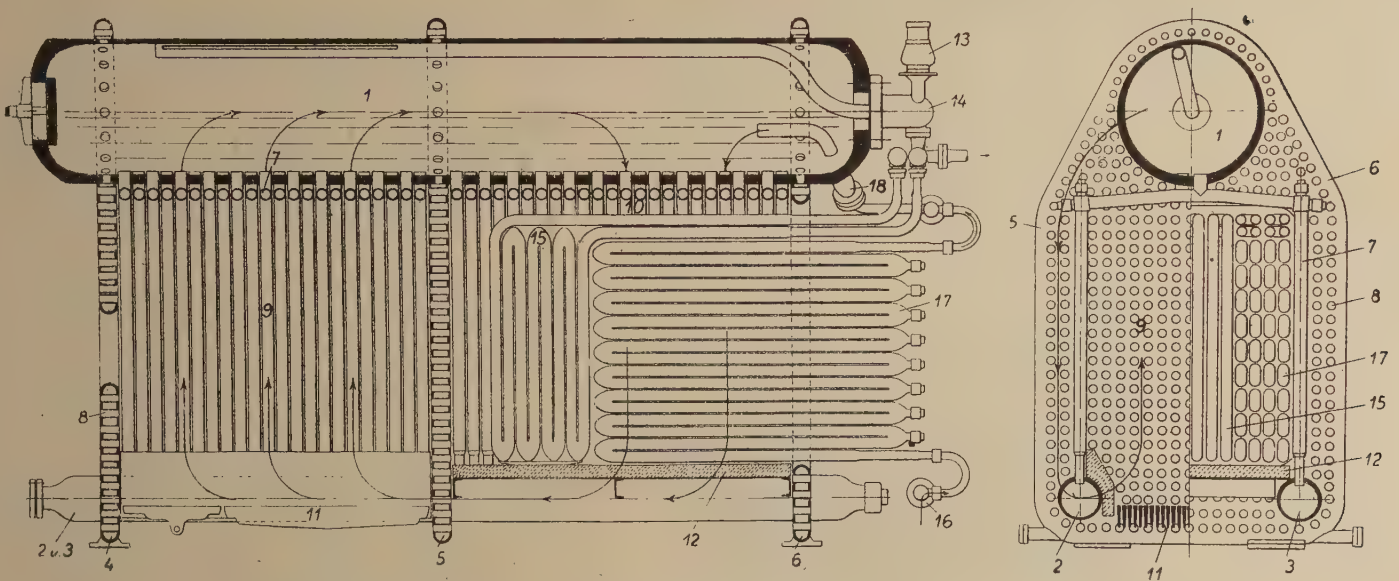


Fig. 6. — Longitudinal and cross sections of the Winterthur boiler. — Scale 1 : 40.

EXPLANATION :

1 = Top drum.
2 and 3 = Bottom drums.
4 = Back water wall of the firebox.
5 = Front water wall of the firebox.
6 = Front water wall of the boiler.
7 = Tubes.
8 = Tubular stays.
9 = Firebox.
10 = Superheater and preheater chamber.
11 = Grate
12 = Firebrick lining.
13 = Safety valve.
14 = Regulator.
15 = Superheater.
16 = Feed pipe connection.
17 = Combustion-gas feed water heater or economiser.
18 = Clack box.

S. L. M. therefore abandoned the customary method of construction and followed absolutely new lines when developing the high pressure boiler.

The principal constituent elements of the new boiler (fig. 6) are the large top drum 1, the two lower drums 2 and 3, the cross water walls 4, 5 and 6, and the water tubes 7. The drums in S. M. steel are forged in one piece, and in order to cheapen the cost are turned eccentrically; it is only at the part where the water tubes are inserted that the full wall thickness of 46 and 26 mm. (1 13/16 and 1 1/32 inches) has been maintained. The cross water walls surround the drums to which they are electrically welded inside and outside. Small diameter hollow stays hold together the two plates of each water wall; a part of the tubes being arranged round the drums make it possible to use a seam welded inside and to lighten the strain on the welds themselves by small stiffening tubes, as well as to allow the welds to be inspected from inside before these tubes are inserted (fig. 7).

The two 11 mm. (7/16 inch) plates of the cross water walls are flanged at their outer edges, welded together inside and out, and reinforced by Höhn strap plates. These are the only welded joints on the boiler subjected to tension : in all the other assemblies, welding has only been used to ensure the joints being tight. Various parts prepared in accordance with the method given above, which were tested before the water containers were finally assembled together, shewed that the welds could stand pressures of 500 kgr. per cm^2 (7 100 lb. per square inch) without cracks or leaks occurring. As a precautionary measure, the elements have also been subjected to a steam pressure test of 200 kgr. per cm^2 (2 840 lb. per square inch) followed by sudden cooling down with cold water. Even after this very searching test, there were no defects. It should be noted, in addition, that the welds subjected to tension mentioned above are not subjected to radiations from the fire nor scrubbed by the combustion gases, so that there is no reason for any anxiety as regards safety. The distribution of the tubular stays over the remaining part of the transverse water walls has to be decided in accordance with the pressure of the boiler. These small tubes are screwed into the walls, then expanded, and protected by a special method of welding against leaking. The sides of the firebox and of the superheater chamber are composed of tubes known as vaporisers in the form of an inverted U and reduced in size for a length of 250 mm. (10 inches) at their lower ends so as to make it easier to renew them. The ends (fig. 8) are cut out of the solid, are fitted with inspection caps, are fastened to the tubes by screwed connections and made tight by welding. The caps themselves are fitted with special joints which have given good results. The space between the vaporising tubes is 5 mm. (0.197 inch) filled in by fusible glass and asbestos.

Experience has shewn that the natural circulation of the water indicated by the arrows in figure 6 is very active, and entirely in agreement with the preliminary tests made with a small scale model in glass, identical in design as regards principle.

Figure 9 represents the finished boiler without its lagging : its robust construction will be noticed, also the inspection and wash-out plugs on the cross water walls, and, in the back water wall, the fire-hole opening. Figure 10 shews the construction of the front water wall; it has a large opening through which the superheater and economiser are got into place. It also carries on the outside a frame of flat bars by which the smoke box, in heavy plate, is connected to the boiler. The smoke box and the front cross water wall are rigidly connected with the frame of the locomotive, whilst the back water wall is carried on rollers so that the boiler can freely expand backwards.

Figures 11 and 12 shew the superheater and combustion gas feed water heater or economiser. The drawn tubes are of S. M. steel; they are 5 and 4 mm. (0.197 and 0.157 inch) thick. The back ends of the feed water heater tubes are

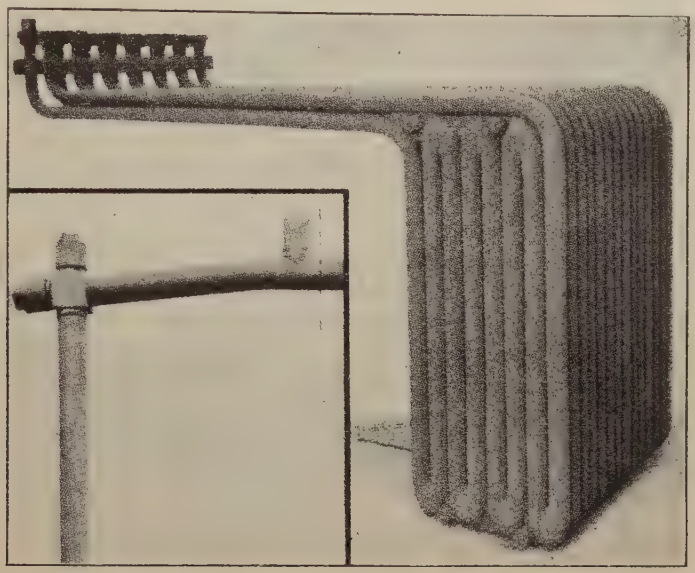


Fig. 8.
Part of a tube element.

Fig. 11.
Superheater.

fitted with screwed plugs readily removed to allow the tubes to be cleaned mechanically. Tightness is secured by special joints which can be used again several times when the nuts are removed. A small air or steam operated mechanical tube cleaner is used to clean both straight and bent tubes so that all scale can be quickly removed. The ordinary washing out of the heater is done by introducing water under pressure into the nest of tubes, the upper bent tubes of the economiser having been removed beforehand.

Steam engine. — The selection of the most suitable engine for this type of locomotive presents many difficulties. As we considered that the use of condensing did not lend itself to the traction conditions of railways, the turbine did not come into consideration. In a general way, this kind of locomotive does not respond to great fluctuations of tractive power. The lack of success of the turbine locomotive in practice so far must be ascribed to this reason : tests are being made to improve the power curve by using different gear ratios, but every improvement so obtained is reflected in increased production costs. In accordance with the principle that in locomotive practice only simple construction survives in the long run, we selected after very thorough examination of all the solutions possible, a uniflow three-cylinder, high speed, double acting engine, the whole of which (fig. 13) can be attached to the frame in a very flexible manner. The choice of this type of en-

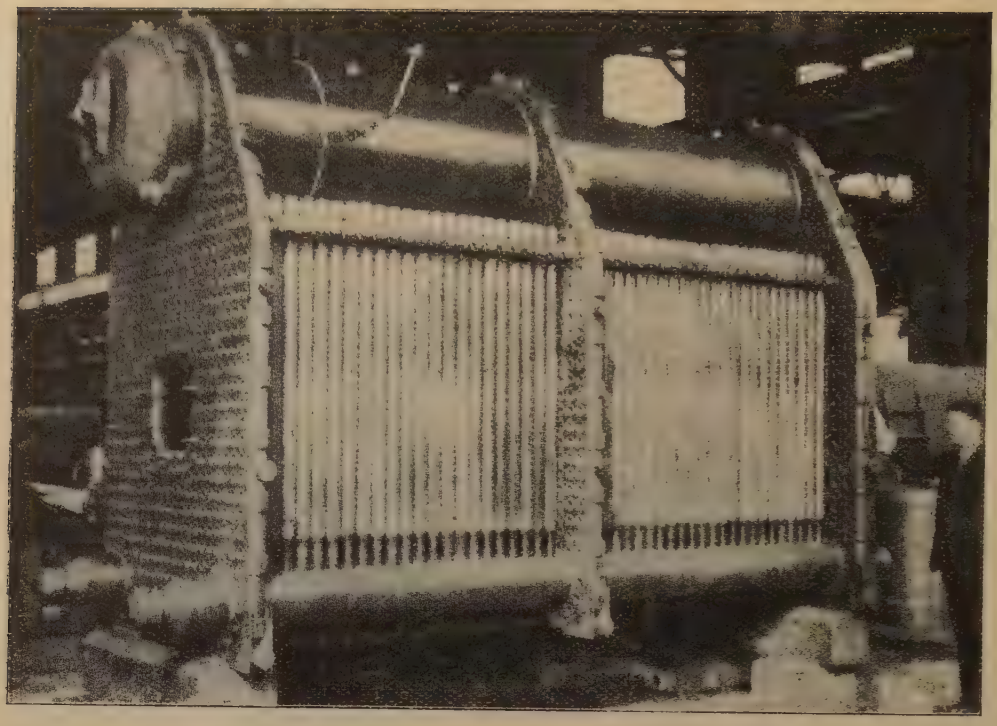


Fig. 9 — Side and back view of the boiler.

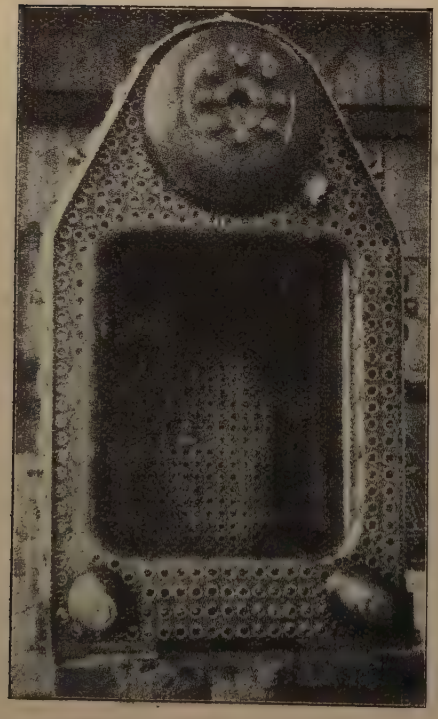


Fig 10. — Front view of the boiler.

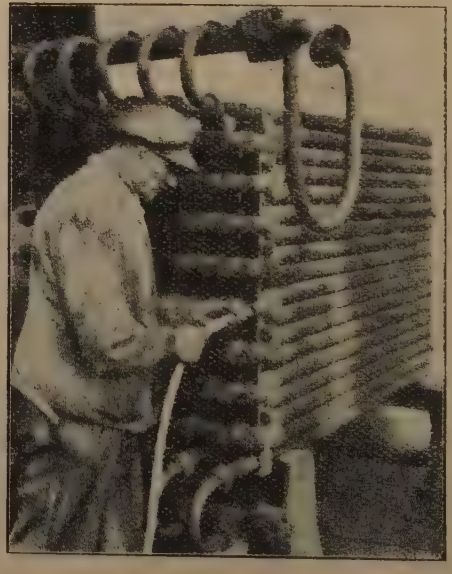


Fig. 12. — Cleaning the economiser.

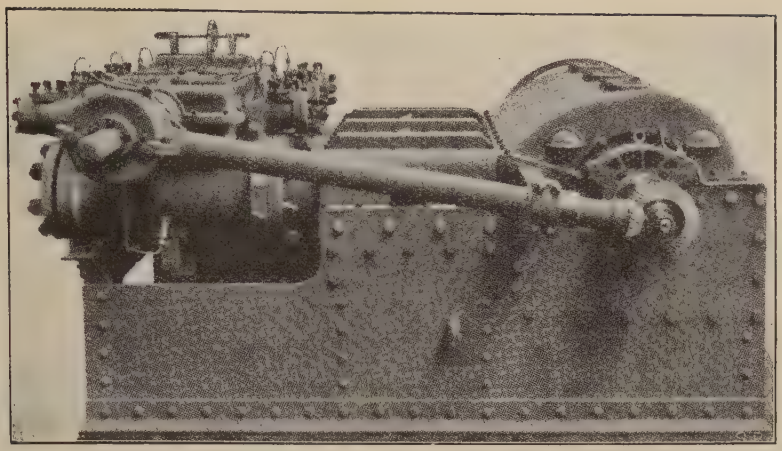


Fig. 13. — General view of the high pressure steam engine.

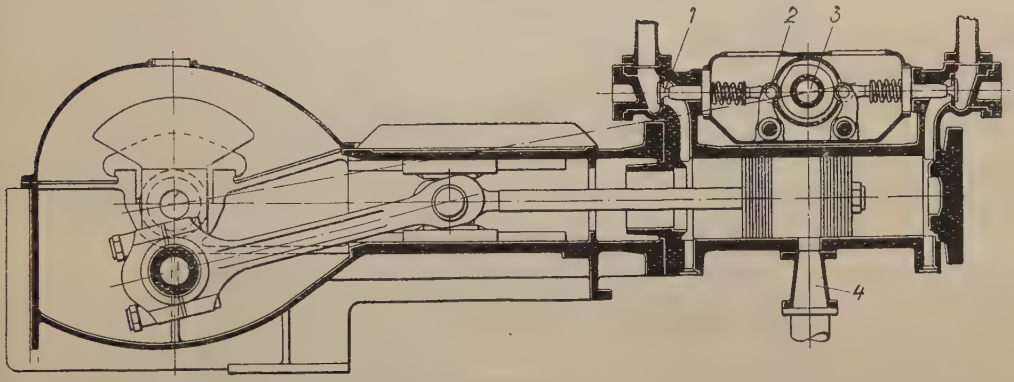


Fig. 14. — Longitudinal section through the steam engine. — Scale 1 : 20.

EXPLANATION : 1 = Admission valve. — 2 = Cam rocker. — 3 = Cam shaft. — 4 = Exhaust pipe.

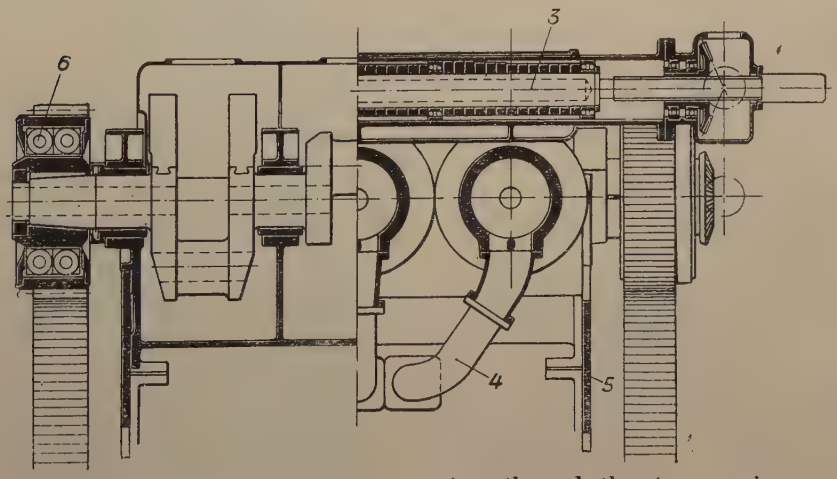


Fig. 15. — Diagrammatic cross sections through the steam engine.

EXPLANATION : 3 and 4, see figure 14. — 5 = Frame plates of the locomotive. — 6 = Springs in the pinion wheel.

gine was partly inspired by automobile practice where multiple cylinder motors are being increasingly used. The convenient arrangement obtained by placing the motor high up at the forward end enabled the number of cylinders to be increased to six. The engine of the « Winterthur » locomotive is designed to give 1 000 brake horse power measured on the crank shaft. For a short period it can be increased to 1 500. To develop this power, three cylinders working in parallel are sufficient. The cylinder diameter is 215 mm. (8 1/2 inches) and the stroke 350 mm. (13 3/4 inches).

The layout of the engine and the valve gear is shewn in figures 14 and 15. Each cylinder has two valve boxes cast integrally with it. The compression being as high as 35 atmospheres per cm^2 (500 lb. per square inch), single seated valves can be used, the seat being only

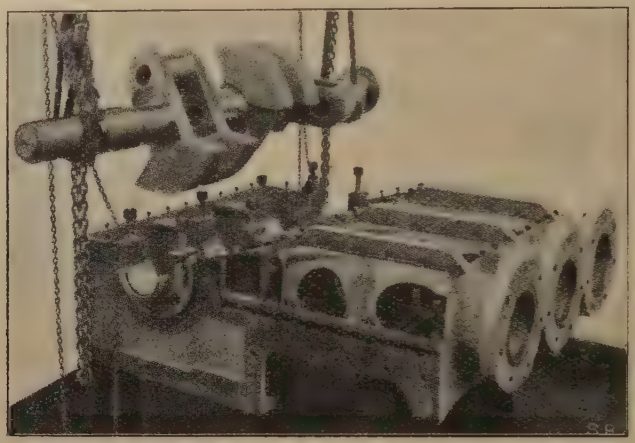
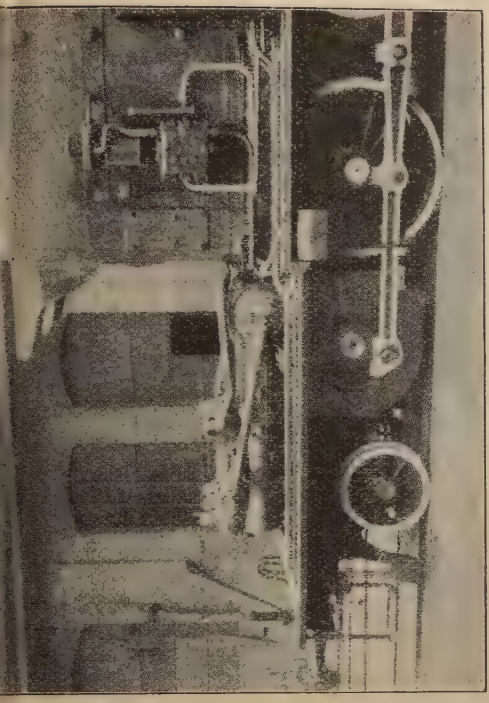
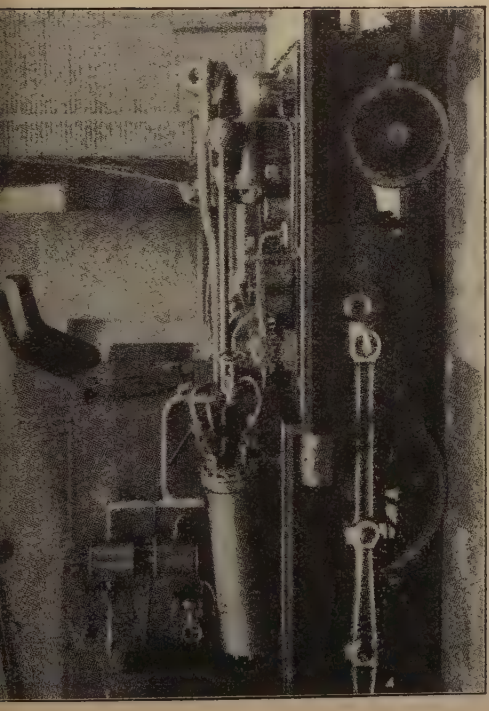


Fig. 16. — Frame of the engine with the crankshaft lifted clear.

50 mm. (2 inches) across. The exhaust is controlled by the piston itself with a pre-exhaust of 15 %. A cam shaft, having six forward and six reverse cuts-off, operates the valves. The change of cut-off is made by steps and not continuously : the tests in service have shewn that this caused not the slightest inconvenience in working. The change in cut-off is made by the gradation of the length and the height of the cams. In order to make it possible under all working conditions to ensure the definitive movement of the cam shaft, the intermediate rockers placed between the valve spindles and the cams are held by lateral springs in the mid position; when, for example, the cam shaft is moved to increase the rate of cut-off, the pressure springs are compressed so long as the movement of the roller is prevented by the longer cam of the next higher cut-off. As each cam only acts on a relatively short part of the face of the roller, the movement, assisted by the springs, can be made even at the highest speeds with the minimum of effort at the reversing wheel. The change of cut-off in the opposite direction would of course be possible without the centering springs. To alter the rate of cut-off with certitude whilst the engine is standing, a compressed air device has been introduced which raises the valves off their seats and brings the rollers far enough back for the cam shaft to be moved into each position, without there being contact between the cam and the roller. Use is





Figs. 17 and 18. — Front of the locomotive with the casing removed.

also made of this arrangement when running with the regulator shut in order to protect the valves and also to reduce the resistance of the locomotive to a minimum when so running. It should be noted at this point that the resistance of the high pressure locomotive running with the regulator closed is much less than that of the piston locomotive of the usual type.

The other parts of the locomotive do not call for any special comments : they are of the usual type adapted to the power to be transmitted. The gears are based on the well tried models used on electric locomotives. The pinions themselves are fitted with a spring drive so as to distribute equally between the two sides the transmission of the turning moment, and also to take up any irregularities in the tangential thrust of the engine as far as the springs are able to ensure regularity in the drive. The piston rod gland packings have so far given satisfaction : they are made up of segments in three pieces which are held against the piston rods by springs and can be replaced at any time without removing the piston rod itself. The details of the motion are enclosed and run in an oil bath. In addition, a hood in steel plate covers in the whole of the front part of the locomotive and protects the engine, etc., against the weather. A Friedmann lubricator supplies oil to the parts of the machine which work under pressure, and two gear pumps force the lubricating oil through the bearings and details of the motion. The oil is carefully filtered and cooled.

Figures 16 to 18 shew the engine itself and the way it is carried on the frame of the locomotive.

Auxiliaries. — To-day the difficulties formerly experienced with the safety valves and water level indicators have been overcome. It is true that even with the very robust Klinger water gauges the glasses leak after a time, but

it is always due to defective packings or to the flaking away of the glass at the glands, which results in transverse cracks in the glass itself. Burst glasses, fragments of which might hurt the men, have not been experienced. An apparatus that has never got out of order, and has always worked properly, is the water level indicator with a float used ever since the first tests. The *feed pump* supplied by the Knorr Brake Company now works satisfactorily after many alterations. The compressed air pump and the steam heating equipment, supplied through a reducing valve are of the normal types. The high pressure valves and cocks were made in our own works and have given no trouble in service.

Tests. — For more than a year the boiler and the engine were subjected to very thorough stationary tests, and were altered as found necessary. In November 1927, the running trials were begun. Thanks to the care with which the preparatory work had been done, all the service tests were carried through without trouble and the locomotive is now in regular working on the Swiss Federal Railways.

Test carried out the 3 August 1927.

Actual duration of the test.	1 h. 20 m.
Coal used	890 kgr. (1 962 lb.).
Water used	5 970 kgr. (1 314 British gallons).
Average horse power at hour rating.	1 126
Coal used per horse-power hour	0.79 kgr. (1.74 lb.).

I. — *Temperature of the water and of the steam :*

t_{w_1} = Temperature of the water before the feed pump;
t_{w_2} = — — — — economiser;
t_{w_3} = — — — after the economiser;
t_{d_1} = — — saturated steam;
t_{d_2} = — — superheat.

II. — *Temperature of the atmosphere and of the combustion gases :*

t_1 = Temperature of the outside air;
t_2 = — — air under the grate;
t_{g_1} = Temperature of the combustion gases before the superheater;
t_{g_2} = Temperature of the combustion gases after the superheater;
t_{g_3} = Temperature of the combustion gases after the economiser;
t_{g_4} = Temperature of the combustion gases in the chimney.

III. — *Steam pressure; number of revolutions and power :*

P_M = Pressure in the steam chest;
p_K = — — boiler;
n = Number of revolutions per minute;
N_e = Effective horse power.

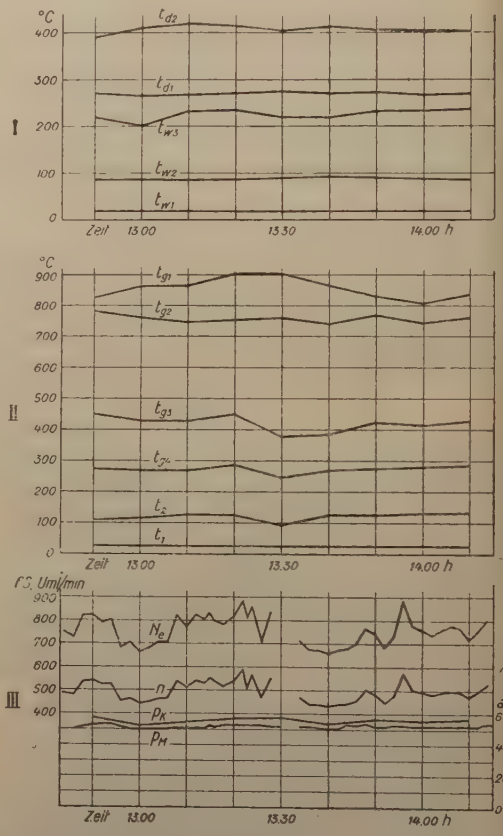


Fig. 19. — Stationary tests of the engine.

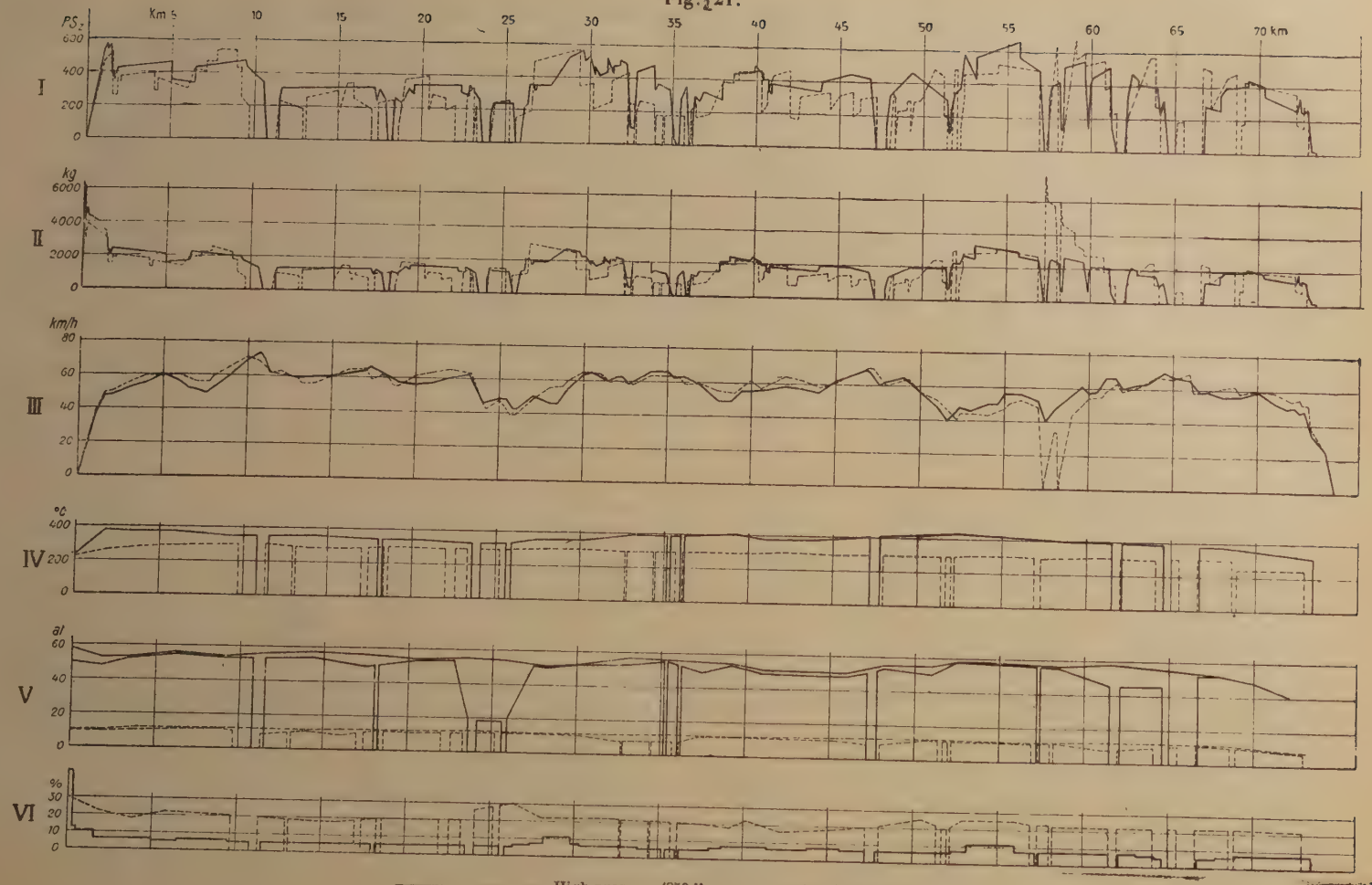
Explanation of German terms : P. S. = Horsepower. — Uml./min. = Revolutions per minute. — at. = kgr. per square centimetre.

Tests, some lasting eight hours, have been made under the most varied conditions. Figure 21 gives the results of a series of tests; all data relating to the boiler and engine will be found therein. The high efficiency (see fig. 5) is principally due to the well chosen arrangement of superheater and combustion gas feed water heater. The consumption of coal recorded during the stationary tests varied according to the load between 0.6 and 0.8 kgr. (1.32 and 1.76 lb.) per effective horse power at the crankshaft and the steam consumption including all auxiliaries between 5 and 6 kgr. (11 and 13.2 lb.). The power was measured by a hydraulic brake. The results have been confirmed by tests made by the « Association des propriétaires suisses de chaudières à vapeur ».

As is well known, the consumption of coal and water of locomotives alters appreciably under service conditions. In order to ascertain these figures as well, in January 1928, comparative tests were made between the high pressure locomotive and a low pressure superheated locomotive of equal power, having a boiler pressure of 12 kgr. per cm² (170 lb. per square inch). The leading dimensions of the two engines shewn alongside in figure 20 are given in the following table :

—	« *Winterthur* ». *high pressure locomotive.*	*Low pressure locomotive.*
Pressure	60 kgr. per cm² (850 lb. per square inch).	12 kgr. per cm² (170 lb. per square inch).
Grate area	1.33 m² (14.30 square feet).	2.30 m² (24.75 square feet).
Heating surface, water side	97 m² (1 044 square feet).	120 m² (1 292 square feet).
Superheating surface	20 m² (215 square feet).	32.20 m² (347 square feet).
Water in the boiler	2.7 m³ (594 British gallons).	4.90 m³ (1 078 British gallons).
Number of cylinders	3	2
Diameter of cylinders	215 mm. (8 1/2 inches).	540 mm. (21 1/4 inches).
Piston stroke	350 mm. (13 13/16 inches).	600 mm. (23 5/8 inches).
Gear ratio	1 : 2.5	...
Diameter of driving wheels	1 520 m (4 ft. 11 13/16 in.).	1 520 m. (4 ft. 11 13/16 in.).
Maximum speed, per hour	75 km. (46.6 miles).	75 km. (46.6 miles).
Weight empty (tender included in the case of low pressure engine)	62.8 t. (61.8 English tons).	64.2 t. (63.2 English tons).
Weight in working order	75.0 t. (73.8 English tons).	90.8 t. (89.4 English tons).
Water	6.2 m³ (1 365 British gallons).	16 m³ (3 520 British gallons).
Coal	2.7 t. (2.66 English tons).	4 t. (3.93 English tons).

Fig. 121.



——— High pressure (850 lb. per square inch) locomotive 101.

. . . . . Superheated locomotive (170 lb. per square inch) B 3/4.

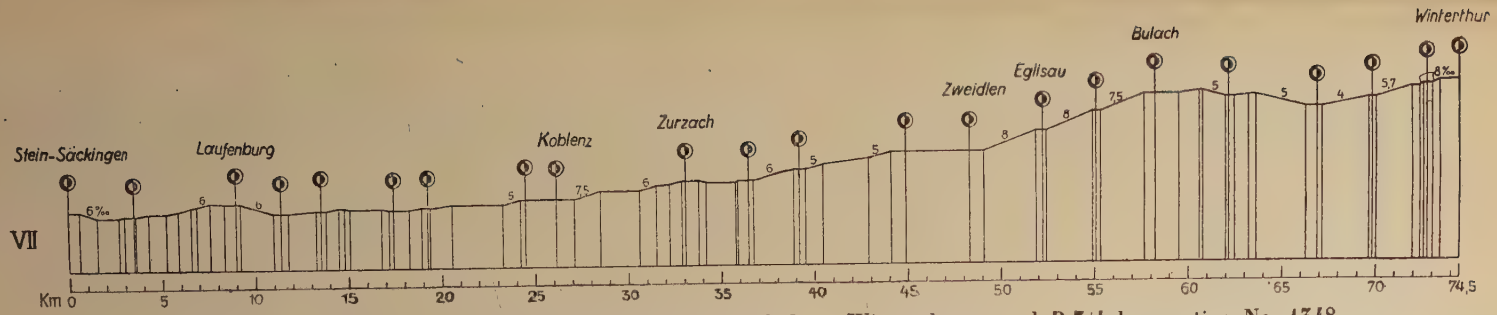


Fig. 21. — Section over which comparative trials of the « Winterthur » and B 3/4 locomotive No. 1348 of the Swiss Federal Railways were carried out.

EXPLANATION :

I = Power developed at the drawbar in horse power. — II = Drawbar pull in kilogrammes. — III = Speed in kilometres per hour. — IV = Superheat, degrees C.
V = Pressure in the boiler and in the steam chest in kgr. per cm². — VI = Cut off, per cent. — VII = Profile of the Stein-Winterthur line.

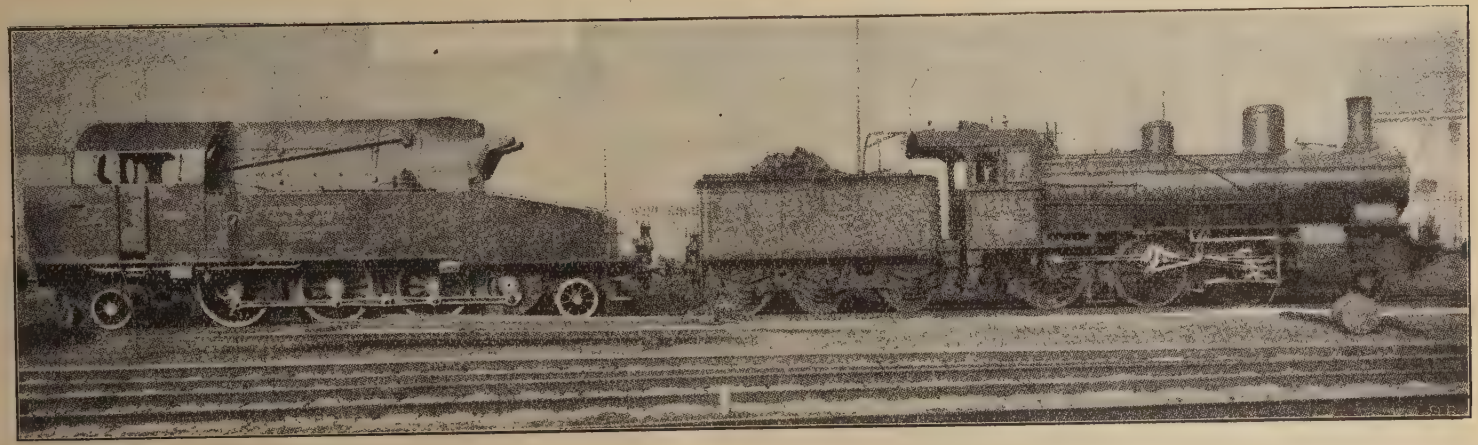


Fig. 20. — « Winterthur » high pressure (850 lb. per square inch) locomotive and the simple expansion superheater (170 lb. per square inch) B 3/4 locomotive of the Swiss Federal Railways.

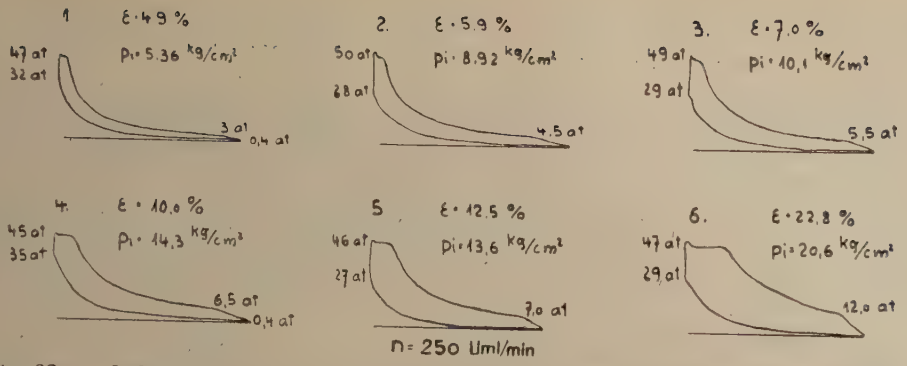


Fig. 22. — Indicator diagrams from the 850 lb. per square inch pressure steam engine.

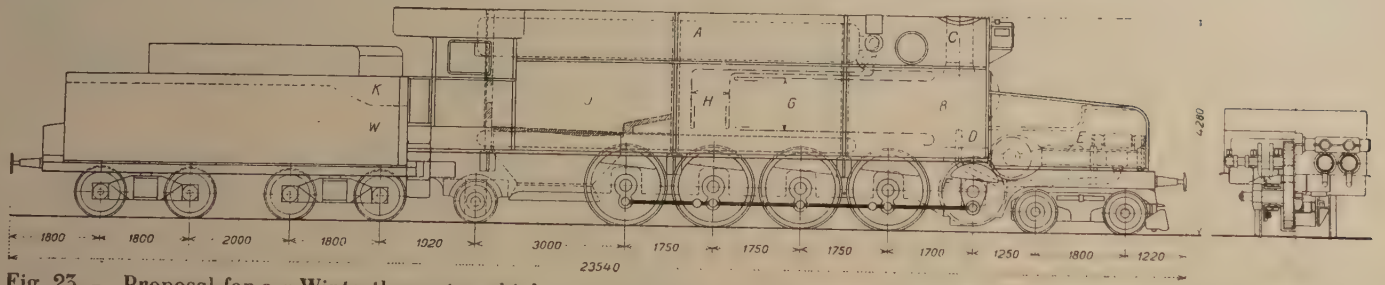


Fig. 23. — Proposal for a « Winterthur » type high pressure locomotive to develop 2 000 effective horse power at the crankshaft.

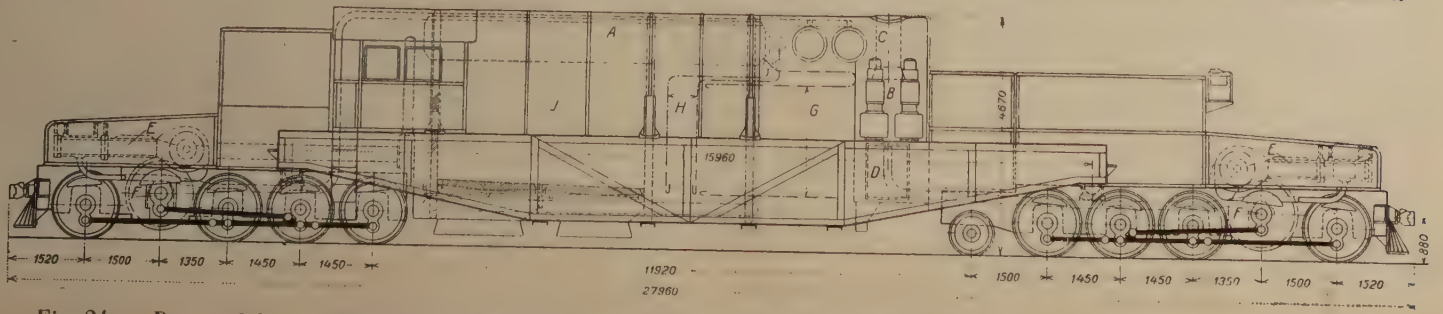


Fig. 24. — Proposal for a « Winterthur » type high pressure locomotive to develop 4 000 effective horse power at the crankshaft.

The tests were carried out over two consecutive days under the same weather conditions with the same train formation on the Winterthur to Romanshorn and Winterthur to Stein-Säckingen lines. Figure 21 reproduces the results recorded by the instruments in the dynamometer car belonging to the Swiss Federal Railways.

During the test runs the following figures were obtained :

—	*Winterthur to Romanshorn and back.*		*Winterthur to Stein-Säckingen and back.*	
Length	112 km. (69.6 miles).		149 km. (92.6 miles).	
Maximum gradient	12 mm. per metre (1 in 83).		8 mm. per metre (1 in 125).	
Load hauled	242 t. (238.2 English tons).		300 t. (295.3 English tons).	
Number of axles	31		40	
	High pressure.	Low pressure.	High pressure.	Low pressure.
Average speed, per hour	61.8 km. (38.4 miles).	60.7 km. (37.7 miles).	55 km. (34.2 miles).	53.5 km. (33.2 miles).
Coal used	776 kgr. (1 710 lb.).	1 176 kgr. (2 592 lb.)	1 012 kg. (2 231 lb.)	1 449 kgr. (3 194 lb.)
Water used	5 250 l. (1 155 British gallons).	9 700 l. (2 135 British gallons).	6 550 l. (1 442 British gallons).	12 200 l. (2 685 British gallons).

If the consumption per draw-bar horse-power-hour developed is compared, the high pressure engine will be seen to have given a *saving of 35 to 40 % of fuel* and *47 to 55 % of water.*

Consequently the « Winterthur » locomotive has fully justified expectations.

Figure 22 is an exact reproduction of the indicator diagrams for different degrees of cut-off. The cut-off varies between 4.9 and 22.8 per cent. The corresponding admission line is nearly horizontal, especially in diagrams 4 to 6, and shews the quick cut-off. The opening to exhaust is equally well defined : in general, the valve events are very good. The steam pressure at exhaust for the principal running notches 1 to 4 varies between 3 and 6.5 kgr. per cm² (43 and 92 lb. per square inch), actual pressure. It is relatively high. but this is necessary to get sufficient vacuum in the smoke box. By adding low pressure cylinders, it would be possible to carry the expansion still farther, and thereby obtain a further coal and water saving; but this would hardly be sufficient to make good the extra cost involved.

The interior and the exterior cleaning of the boiler is easy and can be done very thoroughly. The scale is deposited in the feed water heaters : in the boiler itself, only fine mud in small quantities is found. The cross water legs have not shewn the slightest traces of scale. It is therefore to be expected that the washing out of the new boiler will be much less costly than in the case of the ordinary type.

The relatively small quantity of water in the boiler, roughly half that in a boiler of the usual pattern, might lead one to fear that the heat storage would be too small for working requirements. Actually, however, the tests in service have shewn these fears to be unjustified: the high pressure is, on the contrary, extraordinary flexible, and the effect of

firing is surprisingly rapid, so that it is possible to run with a relatively low fire and adjust the pressure to suit the running conditions. The storage capacity is represented by a greater quantity of steam and by the variation in pressure in the boiler.

Raising steam requires only one and a half hours, or half time and consequently half the coal needed for the low pressure locomotive.

The « Winterthur » high pressure locomotive frankly can be considered as a step forward in the development of the steam locomotive. If this first example, as is natural enough, cannot be qualified as absolutely perfect in every respect, the calculated consumption figures have been confirmed by it, and subsequent designs will be based on solid foundations on which we can confidently expect a rapid development will take place without delay.

The method of construction of the high pressure « Winterthur » locomotive is especially suitable for high powers. The arrangement of the firebox enables the direct heating surface to be given much larger dimensions than in the case of the usual type. Furthermore, for the same reason, the way is opened to the use of pulverised fuel. The diagram of figure 23 shews an express locomotive able to develop 2 000 effective horsepower at the crank shaft. The Garratt type (fig. 24) is fitted with the same engine and develops 4 000 H. P.

[656 .1 (.42) & 656 .2 (.42)]

Road and rail transport [1]

(*The Railway Gazette.*)

In opening his paper, Dr. Fenelon said that it was generally felt that the transport situation in Great Britain was in many ways unsatisfactory, and that the best use was not being made of the facilities available. The problem was a complex and difficult one, but two factors might be singled out as being of especial importance. These were: 1. the rise of road transport and 2. railway traffic losses due to trade depression.

The railways were the backbone of inland transport in Great Britain, and in consequence this depression was a matter of grave importance to the community. The railways had been especially hard hit by the depression in the coal, iron and steel trades, and the importance of this to the railways would be realised when it was stated that even in 1927 coal traffic constituted 50.25 % of the total freight ton-miles carried by the railways. A revival in the heavy industries would do a great deal to improve railway finances.

Road competition.

The railways had also lost traffic to the roads. Though road transport was complementary to, as well as competitive

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[1] Sommary of a Paper read by Dr. K. G. Fenelon before the Economic Section of the British Association, Glasgow, 7 September, 1928.

with, the railways, it was clear that on balance the railways had been the losers.

The growth of motor transport since the war had been remarkable. In 1919 there were only about 250 000 motor vehicles on the roads. In 1920 there were half a million, in 1923 one million, and in 1925 one and a half millions, while this year there would be some two millions. Road transport competition with the railways, moreover, was not confined to this country, but was now general throughout the world.

Loss of traffic was a serious matter to the railways because the industry was one in which fixed charges bore a very high proportion to the total costs of carriage. If traffic declined the average cost per unit of traffic would increase. This argument would apply if the railways were to divert traffic to their own road motors, and herein lies a danger in the unwise use of motor transport by the railways. Road powers for railways will not improve railway finances if, as a result of diversion of traffic to railway-owned road motors, there is not sufficient to keep railway capital adequately employed. Road powers, however, as will be shown later, should, if wisely used, enable the railways to introduce certain operating economies without diverting an undue amount of traffic from the rail. The railway companies also are of the opinion that road powers will provide them with an important bargaining instrument in their negotiations with road transport concerns.

Another difficulty confronting British railways has been the inelasticity of the wages bill. Wages are 140 % higher than they were in 1913, and earnings of railwaymen are even higher owing to more favourable conditions, *e. g.*, overtime, standardisation of grades, etc.

Discussing whether road competition with rail was fair, Dr. Fenelon stated the railway case clearly and forcibly, and came to the conclusion that, on an impartial examination of the arguments on both sides, the railways had on grounds of equity a very good case in their application for extended road powers.

Economic spheres of road and rail.

Road transport is best suited to short-haul, high-grade traffic. Road hauliers who generally base their charges on the cost of service can compete with the railways most easily in the case of high-grade articles, as the railways, broadly speaking, base their charges on the value of the commodity carried.

Generally speaking, rail transport is unrivalled for long-distance journeys, for the carriage of the raw materials of industry and of nearly all kinds of heavy and bulky articles, especially those which pass in quantity. In certain circumstances direct services can be given as from private siding to private siding or from pit to port, and in such cases the economic advantages of rail transport are at a maximum. Where rapid transit is essential, as in the carriage of mails, fish, newspapers and parcels, traffic statistics show that the railways are holding their own with road transport.

Road transport is not in a position to handle heavy traffic, nor to deal with long-distance traffic except in a few cases. Neither can it deal with intensive passenger traffic. In rail transport costs decrease as the amount of the traffic handled increases, and as a result railways are unrivalled for the carriage of heavy traffic. Speed on the rail is greater than on the roads unless the average rate is reduced by terminal, transhipment or other delays. The advantage of speed is one which the railways in all probability will be forced to exploit to the maximum, and the tendency, no doubt, will be to speed-up freight trains. Where road transport is met on its own ground by the provision of special facilities and fast services, the railways have been able to retain or even regain traffic.

An important advantage possessed by the railways is that of their widespread

organisation for the collection and distribution of their traffic. This should be an important factor in any scheme of co-ordination between road and rail transport which might be evolved, and it should prove to be a valuable instrument in carrying out negotiations with independent road transport concerns.

Co-ordination.

As a possible solution of the controversy there had been an increasing tendency to search for some compromise whereby the two methods could be co-ordinated to their mutual advantage and to the benefit of the public. Competition has not furnished a solution, and experience had shown that unnecessary duplications of services existed.

If co-operation could be substituted for competition, the two methods could be used in conjunction and arrangements could be made whereby each system would operate under conditions most suitable to its economic characteristics.

With a co-ordinated transport system it would be logical to treat each natural traffic area as one unit, feeder and development services being operated in conjunction with the main traffic routes of the region. A regular system for effecting transference between road and rail could be introduced, and for this purpose railway stations or depots might be used so that the one organisation would suffice to act as receivers of traffic for both types of transport and thus unnecessary capital expenditure would be avoided. In the future, air transport might be brought into the scheme, and this has, in fact, already been done in Germany.

Co-ordination would prove equally advantageous in many other ways. It would be helpful, for instance, in relieving traffic congestion on a saturated railway line or during periods of seasonal activity. There would then be no need to increase the capacity of the line until traffic had grown sufficiently to justify the great expense of duplicating the tracks or otherwise increasing their capacity. The efficiency of the rail system would be increased if it were possible to utilise road vehicles as « feeders » in areas surrounding the stations. Road vehicles could handle goods traffic to and from intermediate stations more cheaply and more expeditiously than the railway « road vans » or « pick-up goods trains ». The inconvenience, delay and expense of railway transhipment might be minimised or avoided if goods could be brought direct to a central depot by road. In the same way passenger trains could be speeded up and facilities improved. To encourage through traffic of all kinds, inclusive rates and fares could be quoted.

The fact that the various types of transport have each certain advantages and limitations for the carriage of particular types of traffic or for use in certain circumstances provides the economic basis of co-ordination. There is a differentiation in function between each of the principal forms.

There are many practical difficulties in the way of evolving a logical system of co-ordination, and in only one country — Germany — can it be said that a definite attempt has been made to co-ordinate rail, road, air and inland water transport. In Germany, however, road transport is not so developed as in Great Britain or the United States.

The multiplicity of road transport undertakings and the ease with which new enterprises can be started considerably increases the difficulties of inaugurating a system of co-ordination. Co-ordination would be comparatively easy to inaugurate where only railways and large motor transport firms were concerned, but it would be far more difficult to achieve where road transport was in the hands of numerous small owners, some possessing, perhaps, only one or two vehicles.

Co-ordination of road and rail trans-

port might be achieved in a variety of ways, but broadly speaking, the various schemes may be divided into three categories.

Firstly, co-ordination might take the form of voluntary co-operation between road and rail companies (including municipalities), each maintaining their separate identity. In this type of co-ordination, road and rail would be more or less equal partners.

Secondly, co-ordination might be achieved through the provision of road motor services by railway companies or through financial control of road concerns by railways. In this form of co-ordination, road transport would be definitely subsidiary or auxiliary to the railways.

Thirdly, co-ordination might be quasi-legal in character. The scheme put forward by the London Traffic Advisory Committee is of this nature.

Voluntary co-operation would have considerable advantages in that the complementary characteristics of road and rail would be stressed rather than the competitive. Mutual facilities could be provided, such as the use of each other's booking offices for through journeys, use of stations, dovetailing of time-tables, etc. A certain amount of voluntary co-operation between the railways and motor transport has already been attempted in various districts. The London & North Eastern Railway, for instance, works in conjunction with road transport firms at Scarborough, Musselburgh, in the Lake District, and the Trossachs. Such co-operation, however, has been difficult to achieve on an extensive scale.

Railway control of road transport.

In the second type of co-ordination (*i. e.*, railway ownership or control of road transport) the railways would be the predominant partner and road transport would be organised as a subsidiary. The danger of this type of co-ordination is that road transport might become too subordinate, but in practice it would be difficult to establish as vested interests have grown up in road transport. It was partly to effect co-ordination of this form that the railways sought extended road powers, though it is also evident from the proceedings of the Select Committee during the parliamentary inquiry that they hope to use road transport to some extent as a competitive weapon against their road competitors. The railway companies also argued that road powers would put them in a better position to negotiate with road transport firms, and thus road powers might also be used to facilitate co-ordination on a voluntary basis. In Britain, where there are four large railway groups, railway ownership of road vehicles might increase inter-group competition as road vehicles could penetrate each other's territory. This would be contrary to the spirit of the 1921 Railways Act and would make for competition rather than co-ordination. The difficulty might be overcome by the formation of one large road transport company, jointly owned or controlled by the four big railway groups, somewhat in the same manner as the Cheshire Lines Committee is controlled.

From the point of view of railway operating economies, there are many uses to which a subsidiary road transport service could be put by a railway administration. Road services could be substituted for unremunerative branch lines, which in some districts are a serious financial drain on the companies. Circuitous rail routes could be avoided, and other possible economies have already been instanced. Certain experiments have already been made on a small scale in this country, such as the combined road and rail services for agricultural and other produce recently inaugurated in the North of England and Scotland by the London & North Eastern Railway. Steps have also been taken recently by the London & North Eastern Railway and

London Midland & Scottish Railway to increase the use of containers. The Great Western Railway for many years past have used motor lorries to extend their goods services, and they have made considerable use of motor buses. Another example of co-ordination, though not quite on the same footing, is that of the London « Underground » and the London General Omnibus Company at Morden, where a bus service radiates over a wide area from the station.

It is true that certain of the advantages could be obtained by voluntary co-operation between rail and road, but, as has already been indicated, there are certain practical difficulties. The railways, moreover, would seem to prefer to operate or control their own co-ordinated road transport.

Quasi-legal co-ordination

The third type of co-ordination which requires consideration is that of quasi-legal character. In this form the various transport concerns would be required to compose their differences and to evolve a system of co-operative working. Failing voluntary agreement, authority would step in and set up a compulsory scheme of co-ordination. This species of co-ordination would mean the recognition of some form of monopoly, of compulsory participation in the scheme, and of some measure of control in the public interest. A scheme of co-ordination of this type has recently been suggested as a solution of the London traffic problem, and of this the essential characteristics are : 1. common management; 2. pooling of receipts to enable prior charges to be met in all cases; and 3. a controlling body to safeguard the interests of the public. There would be considerable difficulties in extending compulsory co-ordination over the whole country, though the difficulty might be met by the formation of regional traffic authorities who would control transport facilities in their area. These regional committees would, no doubt, be appointed something on the lines of the Electricity Area scheme. The main practical difficulty seems to be that if, on the one hand, a central body were appointed for the whole country, it would be practically impossible to exercise due control on account of the magnitude of the task or to steer clear of the dangers of routine and red tape. On the other hand, if regional committees were set up there would be difficulties due to the fact that the railways transcend local boundaries.

One important advantage of quasi-legal co-ordination over other types is that new road enterprises would not be allowed to run on established routes unless a need for their services could be proved. Under the other forms of co-ordination, a new road transport concern might be able to wreck the scheme of co-ordination by coming in and taking the cream of the traffic.

Limitations of co-ordination from the Public point of view.

All schemes of co-ordination present important limitations, though these vary with the various schemes. The first danger which may lie in co-ordination policies is that a monopoly might be formed which could be exploited against the interests of the public. Competition, though it may lead to unnecessary and wasteful duplication of effort, has certain counteracting advantages from the point of view of the general travelling public and the traders. Under a competitive régime in certain circumstances more frequent services may be given, traders may get more individual attention and passengers may obtain more civility. Competition stimulates enterprise and innovations, while under monopoly there is frequently a tendency to follow routine and for bureaucratic methods to be introduced into the transport organisations. It is therefore more than probable that the evolution of any

scheme of co-ordination on a general scale — whatever be its exact nature — would result in a demand for governmental intervention. Some public body would have to be set up to control the co-ordinated transport in the public interest. The question of public control is a point which will have to be considered by those advocating co-ordination, and the relative advantages and limitations of freedom of enterprise would have to be balanced against those of public regulation.

The indications at present are that some form of co-ordination will have to be adopted. If the various transport interests cannot introduce a system of co-ordination of themselves, then the task will have to be undertaken by an outside body. The demand is most urgent in the case of city transport, and here the probable solution is that of quasi-legal co-ordination, on the basis of regional or rather urban traffic areas. Such co-ordination though it would not solve the road *versus* rail problem, would probably lead to the inclusion of the railways in the city scheme. The wider question of co-ordination throughout the country is more difficult of solution, but there is much to be gained by the public through the adoption of co-ordination, though some method of control is probably essential.

Voluntary co-ordination between the railways and the large road transport companies, which, during the past few years have been tending, especially in the case of passenger traffic, to oust the smaller concerns would be the easiest solution if it can be achieved. To protect the public, in the event of such a combination becoming over-powerful, some form of control would be required. It might suffice if some body such as the Ministry of Transport or the Rates Tribunal were given power to investigate abuses and to shed the light of publicity on any instances of exploitation of the public. If, however, co-ordination cannot be achieved through voluntary endeavour, there would seem to be no other solution than the introduction of some form of quasi-legal co-ordination on the basis of regional traffic committees.

In the subsequent discussion, Sir Josiah Stamp said that from the public point of view it was a good thing that our younger economists should, with fresh minds, untrammelled by past adherence to railway theory, make this question one of impartial study.

MISCELLANEOUS INFORMATION

[625 .143.2 & 625 .143.3]

1. — The steel rail.

Reliability versus wearing capacity.

(*The Railway Engineer.*)

During the discussion which followed Mr. Raymond Carpmael's paper, read before the Institute of Transport recently, some points of considerable interest were raised relative to the reliability of the steel rail. The subject of the paper was *Safety and Speed on the Railways*, and the author included in his appendices a detailed analysis of the causes of railway accidents in Great Britain which had, during the past fifty years, resulted in loss of life, either of passengers or railway servants. Although 14 183 cases of broken rails in running lines had, during this period, been reported to the Board of Trade — or, in more recent years, to the Ministry of Transport, in accordance with the statutory provisions relating to this type of mishap — the appendices above referred to gave the number of lives lost in resulting accidents as only 14 in all, out of a total of 1 475 from accidents of all kinds to trains during this half-century.

High standard of track equipment.

A speaker in the discussion which followed the paper, however, disputed the accuracy of the figure of 14 just mentioned. Five out of the 14 casualties were credited to the period 1910-1914, when the only accident connected in any way with the track, which involved loss of life, was the washing away by a cloudburst of Baddengorm Bridge on the Highland Railway in 1914. Similarly in the 1915-1919 period, to which seven casualties were assigned, the only accident in the track category (apart from the Weedon derailment, of which the initial cause was an engine failure) was the derailment in 1916 of a Midland Scottish express at Little Salkeld, in Westmorland, as the result of a landslide. In neither instance, therefore, were broken rails concerned at all. These were in both cases, indeed, accidents of a type which could not be in any way foreseen, and it is a matter of warm congratulation to all those connected with the maintenance of track in this country that, until the recent unfortunate derailment at Sevenoaks, the permanent way can show so remarkably clean a bill of health in regard to the prime causes of railway accidents. In brief, for a quarter of a century past, not a single railway accident involving loss of life has occurred in Great Britain as a result of defective equipment or maintenance of track.

Wearing capacity of rails.

To this result the soundness of the steel rail has been one of the most essential contributories. Specifications, methods of manufacture, and thoroughness of inspection, both by the works staffs and the railway representa-

tives, have all played their part in producing so excellent a result. No other country in the world, whose railways carry traffic of a density or speed equal to our own, could show a comparable, or anything like comparable, reliability of rails. At the same time it is not wise to lose sight of the relation between this reliability and the wearing capacity of the rails. Is the price that is being paid for perfect soundness in the steel too high ? Is the wearing capacity of the British rail unreasonably low on account of a stringency in limits of analysis or physical tests which the circumstances of the case do not fully demand ? These may be somewhat thankless questions to put, but the importance of the considerations involved, from the economic standpoint, is such that the subject deserves consideration.

It is often claimed that the rails made half-a-century ago in the earliest days of steelmaking on a commercial scale, gave better wear than the rails of to-day. Those making the claim often overlook the difference between the punishment to which the rails had to submit in those days and present-day conditions of wear. In the interim the tonnage of traffic passing over main-line tracks has vastly increased, with a corresponding increase in the abrasive wear to which the rails are subjected. But when this allowance has been made, there is still some force remaining in the claim. Proof is afforded by the fact that early rails have been known to remain in the road — not, of course, in the busiest locations — for periods in excess of 40 years, during which they have been subject to both the old conditions and the new. The explanation of this increased capacity for wear is probably found in the fact that early steel-rail specifications allowed a far wider latitude to the manufacturer than those of to-day; considerably higher percentages of phosphorus, sulphur and manganese were allowed in the steel than would be dreamed of in these days; physical tests were modest in their severity as compared with those now enforced; and, in addition, the rails were rolled from the ingots at lower temperatures than now current, with a result that more work was done upon the steel in reducing it to section. Quite recently, a North of England steelmaker proudly mentionned, in a speech, the fact that a rail made by his firm had remained in the road for 51 years with no greater loss in depth by wear than 1/8 inch. The rail concerned would almost certainly have been rejected on analysis and physical tests under the present British Standard Specification.

British Standard Specification's requirements.

To-day we pride ourselves on the fact that it is possible to demand, and to obtain, drastic limits of purity in the steel employed for the manufacture of rails. Phosphorus, for example, is limited, in the basic open-hearth process, to 0.04 %, and sulphur to 0.05 %. Manganese is cut down to 0.80 % in order to permit of a carbon content ranging between 0.55 and 0.65 %. Yet, from the standpoint of wear, we are no better off than we were decades ago; in all probability we are worse off. The rails of earlier days were without question considerably more brittle than those of to-day, by reason of the greater relative impurity of their composition. But was it a dangerous brittleness ? Surely the accident statistics already mentioned provide the answer. Far into the present century many of these early rails still remained in use, but still without affecting the remarkable British record of freedom from accident through broken rails. Even the thousands of tons of rails produced in Great Britain by the much-maligned basic Bessemer process, deserving though they doubtless were in large measure of the reputation for unreliability which resulted ultimately in the extinction of basic Bessemer working in this country — but not altogether with reason, seeing that, as these columns have proved, it is possible to produce a reliable basic Bessemer steel, giving right methods and raw materials — have not in any way affected the accident record.

It is difficult to avoid the conclusion that considerations of safety have weighed to an undue degree in the framing of the present British Standard Specification to which practically the whole of the rails used by British railways are made. In the drastic purification

of steel necessary in order to eliminate phosphorus, the time of occupation of open-hearth furnaces is being increased, and production costs are forced up. Drastic limitation of sulphur ties down the steelmaker in his choice of iron for the purpose of making rail steel. The ousting of the Bessemer basic process had, rightly or wrongly, done away with the cheapest of all methods of steel production. On the other side of the ledger we have the fact that a certain proportion of phosphorus — within reasonable limits, of course — is of most valuable assistance to rail wear. Equally valuable in its potential resistance to abrasion is manganese, which has been arbitrarily cut down to the figure mentioned in the previous paragraph, to compensate for the doubtful advantages of a higher carbon limit.

The fact has previously been stressed, and deserves yet further emphasis, that British railways purchase between them a quarter of a million tons of steel rails annually, to a total value of two millions sterling. Close attention has been paid to soundness in the steel, and this, coupled with the expert methods of track-laying and maintenance in this country — and particularly the firm support of the bull-head rail in its keyed chair — has produced the successful results in freedom from accident to which reference has already been made. But as to wearing capacity or life, whether of rails, on the track side, or the tyres of the rolling-stock, there is no evidence of any inquiry on the part of the railways commensurate with the economic urgency of the problem.

The following is quoted from an article entitled « Rails and Tyres », which appeared recently in the metallurgical supplement of *The Engineer :*

« This is, surely, a matter of primary im-
« portance to the railways. Abroad it is
« chiefly the railway laboratories that are oc-
« cupying themselves with the difficult prob-
« lem of abrasion testing. Are our railways,
« directly or indirectly, making any efforts in
« this direction ? The user of the material
« generally expects the manufacturer to carry
« out research and to initiate progress. In the
« present instance, however, the gain from
« progress will be entirely with the user — the
« railways. Increased resistance to wear, pro-
« vided without any great additional cost, will
« entail for the steelmaker, at all events, no
« increase of output, but to the railways it
« will be a matter of great financial advant-
« age and even of economic necessity. »

With such a conclusion the writer is in hearty agreement; this is a problem to which the engineering staffs of British railways, in their own interests, should lose no time in directing their energies.

[385 .113 (.52)]

2. — Operating results of the Japanese State Railways for the financial year 1926-1927.

The Japanese Government Railway Department has published the annual report on the working for the year ending 31st March 1927. The English edition came out several months late as it was printed from the original published in the Japanese language.

We believe it will be of value to extract from it the following figures relating to the working of the system. They refer to the Japanese Railways themselves, including the private lines operated by the State, and exclude the lines belonging to the State operated by private companies, these two classes of lines furthermore being of relatively short length.

1. Mileage open to traffic.

Mileage open to traffic.	Average mileage worked.	Mileage open to traffic according to track.					Track mileage.	Number of stations.
		Single.	Double.	Triple.	Quadruple and more.	Total.		
Miles.	Miles.	Miles ch.	Miles ch.	Miles ch.	Miles ch.	Miles ch.	Miles ch.	
7 993.2	7 918.4	6 783 50	1 158 68	12 53	quadruple 49 00 quintuple 0 74 sextuple 1 33 septuple 1 31	8 007 69	13 032 63	2 362

2. Traffic returns.

Traffic.								
Train mileage (1).				Vehicle mileage.				
Passenger trains.	Goods trains	Mixed trains.	Total.	Passenger carriages.	Electric trains	Steam motor cars.	Goods wagons.	Total.
Miles.	Miles.	Miles.	Miles.	Miles	Miles	Miles.	Miles.	Miles.
54 624 341	31 293 347	9 048 685	94 966 373	879 186 686	66 425 266	427 323	1 172 649 983	2 118 689 258

(1) Including the number of miles run by electric trains and steam motor cars computed at the rate of 10 cars = 1 train.

Traffic returns (*Continued*).

Traffic.							
Passengers.							
Number of passengers carried.				Number of passengers carried one mile.			
1st class.	2nd class.	3rd class.	Total.	1st class.	2nd class.	3rd class.	Total.
68 597	17 290 981	718 346 873	735 706 451	20 245 4	722 705 835	11 210 251 698	11 953 203 057

Traffic returns (*continued*).

Traffic.										
Passengers.				Parcels and luggage.			Goods.			
Average journey per passenger.							Tonnage of goods hauled.			
1st class.	2nd class.	3rd class.	Total.	Parcels.	Luggage.	Total.	Ordinary consignment	Express consignment.	Carload consignment.	Total.
Miles.	Miles.	Miles	Miles.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.
295 1	41.8	15.6	16.2	575 665	25 350	601 015	6 414 886	26 633	67 161 246	73 602 765

Traffic returns (*continued*).

Traffic.							
Goods.				Passenger earnings.			
Tonnage of goods hauled one mile.				1st class.	2nd class.	3rd class.	Total.
Ordinary.	Express.	Carload.	Total				
Tons.	Tons.	Tons.	Tons.	£.	£.	£.	£.
755 911 137	6 689 137	6 502 666 192	7 265 266 466	96 899	2 435 187	20 889 194	23 422 280

Traffic returns (*continued*).

Traffic.											
Passenger earnings (*continued*).								Earnings from parcels and luggage.			Earnings from postal matter.
Average per passenger.				Average per passenger per mile.							
1st class.	2nd class.	3rd class.	Total.	1st class.	2nd class.	3rd class.	Total	Parcels.	Luggage.	Total.	
Sh.	Sh.	Sh.	Sh.	Sh.	Sh.	Sh.	Sh.	£.	£.	£	£.
28.24	2.81	0.57	0.63	0.0957	0.0672	0.0371	0.0391	2 366 172	89 055	2 455 227	220 221

Goods earnings.												
Ordinary consignment.	Express consignment.	Carload consignment.	Total.	Average per ton. (1)				Average per ton mile. (1)				Shipping receipts.
				Ordinary.	Carload.	Express.	Total.	Ordinary.	Carload.	Express.	Total.	
£.	£.	£.	£.	Sh.	Sh.	Sh.	Sh.	Sh.	Sh.	Sh.	Sh.	£.
4 521 772	71 798	15 913 464	20 507 034	14.09	4.73	53.92	5.57	1.1963	0.0488	0.2147	0.0565	1 165 886

(1) In computing the average terminal charges, the tonnage of goods of express consignment and of the same hauled one mile are left out of count.

3. Motive power and rolling stock.

Motive power and rolling stock. (1)												
Locomotives.					Passenger carriages.				Goods wagons.				
Tank.	Tender.	Abt system.	Total.	Aggregate weight.	Bogie.	4-wheel.	Total.	Number of seats.	Covered.	Open.	Others (2).	Total.	Aggregate loading capacity.
				Tons.									Tons.
1 158	2 718	89 (3)	3 965	285 988.99	7 775	2 289	10 064	558 605	33 486	28 206	205	61 897	774 748

(1) Includes the vehicles used in construction service.
(2) Includes tank-cars, gas-holders, snow-plows and tender-cars.
(3) Representing electric locomotives.

4. Shipping and transportation.

Shipping.				Transportation.				
Number of ships.	Gross tonnage.	Chartered ships.		Number of engine sheds.	Locomotive mileage.	Vehicle mileage.	Coal consumed for operation.	Oil consumed for operation.
		Number.	Gross tonnage.					
	Tons.		Tons.		Miles.	Miles.	Tons.	British gallons.
61	45 360.72	...	...	119	108 514 494	3 790 927 234	3 106 958	856 607

N. B — Coal consumed for operation includes petroleum dregs converted at the rate of 1 gallon = 14.8 lb. of coal.

5. Finance.

Working expenses.											
Revenue.				Working expenses..							
Passenger traffic.	Goods traffic.	Miscellaneous.	Total.	General.	Maintenance.	Transportation.	Maintenance of rolling stock.	Traffic.	Secret service.	Hotels.	Shipping.
£	£	£	£	£	£	£	£	£	£		
27 783 703	21 015 167	794 326	49 582 196	966 703	5 949 182	7 754 752	2 819 022	8 729 271	4 097	35 983	759 617

Finance (*continued*).

Finance.														
Working expenses (*continued*).				Profit.	Percentage of working expenses to revenue.	Revenue per train mile.	Working expenses per train mile.	Profit per train mile.	Capital.	Construction expenses settled.	Improvement expenses settled.	Additional expenses settled.	Cost of nationalized lines (represented by purchase price).	Percentage of profit on capital.
Rent of private railways.	Subsidies to Relief Association.	Hospital and dressing rooms.	Total.											
£	£	£	£	£	%	Sh.	Sh.	Sh.	£	£	£	£	£	%
...	433 581	288 476	27 740 684	21 841 512	56	10.26	5.86	4.61	275 387 985	4 912 630	15 699 092	557 192	67 403	8.0

N. B. — The construction expenses settled include the expenses incurred by the construction of the Hokkaido lines and a bridge over the new course of the river Yodo.

The construction and improvement expenses settled include advances.

6. Stores.

Stores.					
Railway stores purchased.			Amount of stores on hand.		
Home purchase.	Foreign purchase.	Total.	Stores in stock.	Articles in process of manufacture.	Total.
£	£	£	£	£	£
17 494 745	335 258	17 830 003	2 912 568	38 787	2 951 355

7. Works.

Works.					
Number of workshops.	Number of workmen.	Number of days worked.	Number of days worked adjusted.	Wages.	Average wages per day per capita.
				£	Sh.
21	12 627	3 767 319	3 826 529	1 025 497	5.455

N. B. — Number of workmen represents the average number as returned at the end of each month during the year.
Number of days worked is computed by multiplying number of workmen at work by working days.
Number of days worked adjusted represents the results obtained by counting one day's work at 10 hours, a workman working for 15 hours per day being counted as 1 1/2 days worked.

8. Electricity.

Electricity.								
Number of stations accepting telegrams.	Number of stations accepting general telegrams.	Number of communication apparatus.			Message traffic.			
		Telegraphs.	Telephones.	Block instruments.	Service telegrams.	Public telegrams.	Total.	Shares in charges on general telegrams.
								£
2 403	951	2 737	38 845	4 784	65 048 991	12 755 197	77 804 188	59 722

N. B. — Telegraph apparatus comprises recorders, block instruments and automatic relays.
The block instrument comprises double line instrument, Tyre's Webb Thompson's and single line instruments.
In the returns of message traffic despatched and received messages are counted as one and those transmitted as two.

9. Officials and employees.

Officials and employees.												
Higher grade.						Lower grade.					
Shinnin and *chokunin* rank.			*Sonin* rank.			Clerks of *hannin* rank.			Assistant engineers of *hannin* rank.		
Number.	Monthly salary.	Average salary per capita.	Number.	Monthly salary.	Average salary per capita.	Number.	Monthly salary.	Average salary per capita.	Number.	Monthly salary.	Average salary per capita.
	£.	£ - sh.		£.	£ - sh.		£.	£ - sh.		£.	£ - sh.
22	1 023	46 - 10	823	20 669	25 - 2	11 585	100 146	8 - 13	6 162	58 714	9 - 10

Officials and employees (*continued*).

Officials and employees.								
Employees of *koin* class.			Employees of *yonin* class.			Total.		
Number.	Monthly wages.	Average wages per capita.	Number.	Monthly wages.	Average wages per capita.	Number.	Monthly wages.	Average wages per capita.
	£	£ - sh		£	£ - sh.		£	£ - sh.
(1) 1 776 69 573	(1) 15 334 361 487	5 - 5	110 559	508 412	4 - 12	200 500	1 064 785	5 - 6

(1) Representing railway foremen.

NEW BOOKS AND PUBLICATIONS

[624 .2 (01]

Dynamic stresses and vibrations of the main girders of railway bridges.

Fig. 1, p. 114.

The German review *Die Bautechnik* in its numbers of the 20 and 27 January 1928 published two articles by Professor Hort, of the University of Berlin, on the dynamic stresses in, and the vibrations of, the main girders of railway bridges. This subject, one of those to be considered at the next Congress at Madrid, being of present interest, we give below a resumé of these studies. The author, a distinguished mathematician and the writer of an important work, *Technische Schwingungslehre*, has devoted himself to the mathematical determination and separation of the different elements entering into the composition of the co-efficient of impact or of the dynamic majoration. The calculations only relate to the most usual case of isostatic girders.

We consider, with Professor Hort, that such a mathematical investigation can be most fruitful and ought to be carried much further.

It will not be useless to recall to mind that the calculation of bridges is still actually based *a priori* on purely mathematical data to which practical experience has only made very slight alterations The problem of the impact ought to be treated first of all, and to the widest extent possible, as a mathematical problem. The systematic tests that are being carried out on all the principal railways will shew us the probable corrections required.

The values obtained experimentally *a priori* have shewn in any case by their disjointed and divergent character that it would be well to consider them as having relative values only.

Under these circumstances, every mathematical investigation tending to narrow the problem within closer limits will form a useful contribution to its solution.

One criticism must be made about Professor Hort's work, namely, that he has neglected to point out the very important part the French mathematicians Bresse, Phillips and Renaudot have taken in the study of these questions.

We propose in the near future to deal again with the practical application of the mathematical results mentioned, at the same time bringing out the fact now being established that the French and German formulæ used in calculating the co-efficient of impact are leading to the same numerical results.

I. — Permanent factors acting even in the case of perfectly balanced locomotives.

a) Centrifugal force.

If the track is perfectly regular, the running of the stock taking place without any shock, the motor being for example a perfectly balanced electric locomotive, the deflection of the track and of the bridge

will take place following a curve concave upwards giving rise to supplementary centrifugal forces, which will increase the working stresses.

The English mathematician Stokes has devoted a number of theoretical studies to these investigations. Zimmermann in his pamphlet « The vibration of a girder under the influence of a moving load » has dealt with the problem very completely. The principal practical result can be deduced as follows :

If

$$\varepsilon_z = \frac{1}{\alpha/4 - 3}$$

the dynamic stress will be expressed by :

$$\sigma_{dyn} = \sigma_{stat}\,(1 + \varepsilon_z),$$

α being equal to

$$\alpha = \frac{3}{2} \cdot \frac{ghE}{V^2\,\sigma_{stat}}.$$

This expression becomes by substituting numerical values :

g = the acceleration due to gravity, 981 cm. per second ;
h = 30 cm., depth of girder ;
E = 2×10^6 kgr. per cm² co-efficient of elasticity ;
V = 2 800 cm. per second-speed (100 km. an hour) ;
σ_{stat} static working stress = 600 kgr. per cm² ;
$\varepsilon_z = 0.14$;

with other values :

h = 350 cm.
V = 1 800 cm. per second (65 km. an hour.
$\varepsilon_z = 0.01$.

Zimmerman deduced the following practical result from the integration of the differential equation :

$$\frac{d^2\eta}{d\xi^2} \times \frac{a\eta}{(1 - \xi^2)^2} = 1$$

in which η and ξ are proportional to the ordinate and to the abscissa of the deflected curve of the running surface.

Professor Hort proposes to distinguish this effect from the centrifugal force by calling it the *Zimmermann effect* to which the co-efficient ε_z will apply.

b) **Vibrations of the main girder.**

We can reduce *grosso modo* all types of girders used to plain web girders.

If J is the moment of inertia, f the section. E the modulus of elasticity, ρ the unit mass, and l the span, the vibrations of the girder when free and *not under live load* can be studied by means of the following equation :

$$\rho f dx \frac{d^2y}{dt^2} = dS.$$

S being the shearing stress

$$S = -\frac{dM}{dx} = -EJ\frac{d^3y}{dx^3}.$$

The equation becomes

$$\rho f \frac{d^2y}{dt^2} + EJ\frac{d^4y}{dx^4} = 0.$$

The integral will give the equation of the free vibration of the girder. If the girder is loaded with an uniformly distributed load $p = f(x)$, the equation for the vibrations becomes :

$$\rho f \frac{d^2y}{dt^2} + EJ\frac{d^4y}{dx^4} = f(x) ;$$

these vibrations are superimposed on the curve of the static deflections defined by

$$EJ\frac{d^4y}{dx^4} = f(x).$$

The frequency of the free vibrations of the girder is given by the expression

$$n = \frac{\pi}{2l^2}\sqrt{\frac{EJ}{\rho f}}.$$

In this expression ρf is the mass per unit of length, $\rho f = \frac{p}{g}$ if p is the weight of the girder per metre of length.

If the girder is passed over by a train at a speed V, the vibrations — *forced* — due

to the moving loads can be calculated by the following equation :

$$\rho f \times \frac{d^2y}{dt^2} + \mathrm{EJ}\,\frac{d^4y}{dx^4} = \varphi(t)$$

of the same type as before, the second term being a function of the time.

The practical conclusion to be drawn from this analysis is that the elastic work at the dynamic state under uniformly distributed live load is increased in the proportion

$$\sigma_{dyn} = \sigma_{stat}\,\frac{1}{1 - z^2}$$

$$z = \frac{V}{2ln} \qquad z = \frac{1}{n} \times \frac{2l}{V} = \frac{T_s}{2T_f}$$

in which T_s is the period of vibration of the girder

and T_f is the time required to cross the bridge.

When the movement of an isolated load is in question

$$\sigma_{dyn} = \sigma_{stat}\left(\frac{1}{1 - z^2} + \frac{8}{\pi^2}\,z\right).$$

For a 35 m. (114 ft. 10 in.) span bridge with a frequency of vibration of $n = 4.3$ per second, the speed of movement of a locomotive being 65 km. (40.4 miles) an hour,

$$z = 0.06 \text{ and } \sigma_{dyn} = \sigma_{stat}\,(1 + 0.048).$$

The dynamic majoration of the elastic work is about 5 %.

The dynamic majorations due to these causes which the author proposes to call the *Timoshenko effect* can be expressed by

$$\varepsilon_{T_1} = \frac{1}{1 - z^2} - 1 = \frac{z^2}{1 - z^2}$$

under uniformly distributed load, and

$$\varepsilon_{T_2} = \frac{1}{1 - z^2} + \frac{8}{\pi^2}\,z - 1 = \frac{z^2}{1 - z^2} + \frac{8}{\pi^2}\,z$$

under isolated load.

In the case of a bridge of 100 m. (328 feet) span, the speed being 100 km. (62.1 miles) an hour — $z = 0.36$ (the frequency of vibration of a structure of this kind would be 1.4 Hertz — 1.4 vibrations per second).

$\varepsilon_{T_2} = 0.44$, that is, 44 % of the work under static conditions.

The fact that the period of vibrations proper to the girder is modified by the supplementary mass of the moving load must also be taken into account.

Taking the bridge as being fully loaded, the corresponding period would be T_z. The live load will be sufficiently covered by introducing into the calculations the mean $\frac{1}{2}\,(T_s + T_z)$ instead of T .

c) Vibrations under the influence of the driving wheels.

The vertical component of the centrifugal force due to the excess counterbalance can be represented by

$f \cos wt$ in which w is the angular velocity of the driving wheel,

$w = \frac{V}{R}$, R being the radius of the wheel,

$f = m\,\frac{V^2}{R^2} \times r$, m being the mass of the counterbalance and r the distance of its centre of mass from the centre of the wheel. Taking the cranks as being set at 90° apart on the horizontal centre plane of the engine, f would become $f\sqrt{2}$ for one axle and $jf\sqrt{2}$ for a number j of coupled axles.

The total vertical component of the centrifugal force would be finally :

$$F = j\sqrt{2}\,m\,\frac{V^2}{R^2}\,r \cos wt = f_j \cos wt.$$

In order to bring into the formulæ a factor taking into account the movement of the live loads across the structure, it would be necessary to multiply E by a Fourier series, the general expression for which would be

$$\sin i\pi\,\frac{t}{Tf}.$$

F becomes finally

$$F = f_j \cos wt \sin i\pi \frac{t}{T_f}$$

Substituting

$$z = \frac{T_s}{2T_f} \quad \text{and} \quad \zeta = \frac{T_s}{\frac{2\pi}{w}},$$

$\frac{2\pi}{w} = T_R$ period of rotation of the wheel,

$$\zeta = \frac{T_s}{T_R}.$$

The equation due to Timoshenko resolves the problem in

$$y = \frac{f_j l^3}{EJ\pi^4} \sum_{i=1}^{i=\infty} \left\{ \frac{\sin\left(\frac{i\pi}{T_f} + w\right)}{i^4} \cdots \right\}$$

In the most important case, that of resonance, $\zeta = 1$, $T_R = T_S$, the static deflection stresses due to the force f_j are multiplied by the co-efficient $\frac{0\ 25\pi}{z}$.

For the 35 m. (114 ft. 10 in.) bridge quoted above $z = 0{,}06$, or an increase of from 1 to 13 times the static tension corresponding to the load due to the counterbalance.

If we consider that

$$G \approx 10 \text{ to } 12\, f_j,$$

G being the total weight of the locomotive, this majoration would correspond to an increase of live load of 100 %.

In actual fact, the vibrations are partly damped out, and the theoretical case of resonance rarely occurs.

Furthermore, it must be observed that this increase applies to the locomotive alone, whereas the real live load is produced as much by the wagons as by the locomotive.

If we call this effect — wheel effect or what would be better *counterbalance effect* — the proportional majoration would be

$$\varepsilon_R = \frac{0.25\pi}{z} - 1 = \frac{0.25\pi - z}{z}.$$

The particular effects that we have just examined are to some extent possible of mathematical definition. In addition to these effects, the recorded diagrams of stresses and deflections reveal others in the form of high frequency vibrations which are superimposed upon the general deflections and which represent the *effects of shocks*. These shocks are caused by the irregular movement of the vehicles in motion, due to irregular running surfaces of the rails, to contributing defective tyre surfaces and to nosing movements along the track.

An interesting point of comparison is to be found between these secondary vibrations and those produced in the ground by the movement of tramway vehicles.

If we examine with care a diagram of the deflections, it is possible to recognise the super-imposition of the vibrations due to the various causes.

In the particular case already quoted of a 35 m. (114 ft. 10 in.) bridge crossed over at 65 km. (40.4 miles) per hour, the experimental determination of the different effects) gave the following results :

Zimmermann effect. . . .	$\varepsilon_z = 0.200$
Timoshenko effect	$\varepsilon_{T_0} = 0.088$
Counterbalance effect . .	$\varepsilon_R = 0.112$
Shock effect.	$\varepsilon_{ST} = 0.15$

that is, a total dynamic majoration :

$$\varepsilon_D = 0.55.$$

The theoretical calculations carried out in accordance with the preceding bases had given :

	$\varepsilon_z = 0.01$
	$\varepsilon_{T_2} = 0.05$
	$\varepsilon_R = 1.0$
In all . .	$\varepsilon_D = 1.06.$

The differences are marked. They can be explained in part ; for example ε_z has been calculated by supposing a plain web girder perfectly continuous In actual fact, the girder is a trellis girder, the lower plating of which may be far from being continuous in the neutral plane.

The theoretical and practical values of ε_{T_2} shew closer agreement, which is understandable seeing that the vibratory period of the bridge, as well as the time taken to cross it, can be ascertained with greater precision.

The values for the counterbalance effect ε_R shew more considerable differences : they are due to the fact that we first of all made the assumption of perfect resonance and the absence of all damping out of the vibrations. The theory of the question must be perfected by bringing this damping out of the vibrations into account.

The search for concordance between the calculated and measured effects, and their individual calculation, is the problem to be solved at the present time. To convince oneself of this, all that is needed is to consider the nebulous results of the practical tests carried out on impact, from which so far no defined law has been ascertainable.

Expression of the total dynamic co-efficient.

If we refer to the preceding enquiry, the total dynamic co-efficient

$$\varepsilon_D = \varepsilon_z + \varepsilon_{T_2} + \varepsilon_R$$

$$\varepsilon_D = \frac{1}{\frac{1}{4}\alpha - 3} + \frac{z^2}{1 - z^2} + \frac{8}{\pi^2} z + \frac{0.25\pi - z}{z}$$

$$\alpha = \frac{3}{2} g E \frac{h}{V^2 \sigma} \qquad z = \frac{V}{2ln}$$

This formula can be transposed so as to bring out the effect of the span l, the co-efficients being calculated numerically in accordance with the following suppositions :

$$ln = \frac{13\,750}{\sqrt{\frac{\sigma}{h}}} \qquad h = \varepsilon l \text{ with } \varepsilon = \frac{1}{10}.$$

$E = 2.2 \times 10^6$ kgr. per cm², $g = 981$ cm. per second,

f_j being the maximum centrifugal force of the counterbalance weights and G_a the adhesive weight,

$$\frac{f_j}{G_a} = 0.1$$

$V = 2\,200$ cm. per second,

$\sigma = 1\,000$ kgr. per cm².

ε_D is given finally by

$$\varepsilon_D = \frac{1}{0.017 l - 3} + \frac{64}{l} + \frac{6.4}{\sqrt{l}} + 0.01 \sqrt{l} - 0.1.$$

The value of ε_D calculated in relation to l shews an increase as l becomes greater. The reason is that we have taken the axles as being in perfect resonance. In these conditions, the more the bridge increases in length, the more marked this effect will be.

ε_D also increases with the speed and the degree of stress.

When endeavouring to arrive at the value of the excess counterbalance effect in a way more nearly in conformity with actual conditions the weight of the coupled wheels can be taken as only representing a proportion of the weight of the train, say 1/3 for example.

ε_D then becomes, conserving the preceding numerical values :

$$\varepsilon_D = \frac{1}{0.017 l - 3} + \frac{64}{l} + \frac{6.4}{\sqrt{l}} + \frac{1}{3}(0.01 \sqrt{l} - 0.1).$$

If we add to this curve of ε_D the Saller curve, taking into account the shock effect properly speaking, we get a resulting curve coinciding in a striking manner with the impact curve adopted by the German State Railways for the case of a bridge with the track laid on sleepers carried directly on longitudinal girders.

We reproduce below the curves in question (fig. 1).

Vibrations of the bridges.

If we first of all consider a plain web girder of height h, moment of inertia J, bending modulus W, permanent load p, live load q :

σ_p working tensile stress in the booms under the permanent load, σ_{p+q} under permanent load + live load :

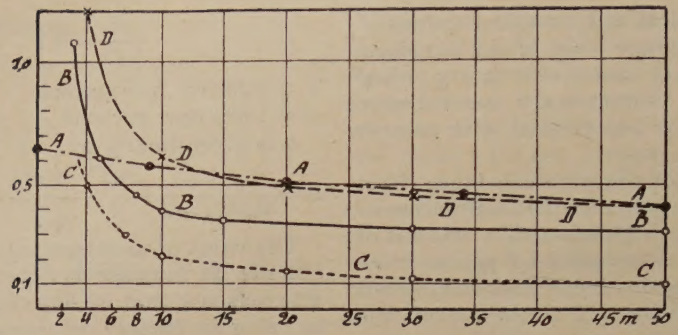

Fig. 1.

Explanation — *AA* : Curve corresponding to the formula of the German State Railways. — *BB* : Theoretical curve — Zimmermann and Timoshenko effects. — *CC* : Saller's curve. — *DD* : Curve resulting from *BB* and *CC*.

the numbers n_p, n_{p+q} giving the numbers of vibrations per second are :

$$n_p = \frac{\pi}{2l^2}\sqrt{\frac{EJg}{p}}; \qquad n_{p+q} = \frac{\pi}{2l^2}\sqrt{\frac{EJg}{p+q}}$$

$$\sigma_p = \frac{pl^2}{8W} \quad \sigma_{p+q} = \frac{(p+q)l^2}{8W}, \quad W = \frac{2J}{h}$$

$$n_p = \frac{\pi}{8}\sqrt{Eg} : l\sqrt{\frac{\sigma_p}{h}}$$

$$n_{p+q} = \frac{\pi}{8}\sqrt{Eg} : l\sqrt{\frac{\sigma_{p+q}}{h}}$$

To sum up :

$$n = \frac{\gamma}{S}, \quad \gamma = \frac{\pi}{8}\sqrt{Eg} = 17\,600$$

$$S = l\sqrt{\frac{\sigma}{h}}$$

After investigating different kinds of girders, including trellis-work girders, and having assimilated them to plain web girders, Mr. Buhler has shewn that we can express n by an average value $= \frac{13\,750}{S}$,

$$\gamma = 13\,750.$$

The number n can be used to diagnose the organic condition of a structure.

$$n = \frac{\pi}{8l}\sqrt{gh}\sqrt{\frac{E}{\sigma}}.$$

The factor $\frac{\pi}{8l}\sqrt{gh}$ is constant as regards time, whereas $\sqrt{\frac{E}{\sigma}}$ decreases as the time ; E will diminish according as the organic condition of the structure becomes more defective, through rusted parts, loose rivets, etc.

σ will vary inversely so that finally $\sqrt{\frac{E}{\sigma}}$ will diminish.

In the course of time, the variations of n can become an index as to the organic condition of the construction.

Principal conclusions.

It is possible to determine theoretically the factors forming the dynamic majoration of the stresses at the middle of the girder. By adding to it the factor of irregular shock as proposed by Mr. H Saller, we obtain a total in practical agreement with the curve the German Railways use in the case of bridges with the track laid on sleepers resting directly on longitudinal girders

In any theoretical investigations carried out in the future, the damping out of the vibrations must be taken into account, as must be the effect of the supports of the bridge on the vibrations.

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